# Finite Element Structural Analysis of Movable Crusher Supports 

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## Summary

Finite element analysis is applied to two different basic structural arrangements. One configuration utilizes a plate shell structure to carry the loads similar to the construction used for the marine industry. The other configuration utilizes a space frame made up of columns, beams and bracing similar to the construction used for buildings. Both static and dynamic analysis are carried out for the two configurations.

## 1. Plate Shell Structure

### 1.1 Geometry

The basic shell structure used to support the crusher and associated equipment is a modified toroid. The modified toroid is a semimonocoque structure characterized by a stiffened plate skin that carries a major part of the loads.
The crusher is supported at the center of the ten foot deep toroid by a series of gusset plates which are welded to the inner ring of the toroid. The toroid is supported on three legs, the bottom approximately twelve feet above the ground. The toroid is fifteen sided on the exterior and approximately 48 ft in diameter. The interior of the toroid is a 21 ft diameter circle. The centers of the legs are on a 58 ft diameter circle.
The upper and lower decks of the toroid consist of floor plate and beams to carry the equipment loads and floor live loads to the toroid walls. The toroid walls act as deep beams to transfer the loads to three shear walls that run radially inside the toroid and continuously down into each support leg.
The legs are box sections. An interior cruciform stiffens the exterior plates against buckling. The three legs rest on rectangular steel footing pads that are placed and leveled prior to the placing of the structure on them. The maximum soil bearing pressure is about $4,000 \mathrm{lbs} / \mathrm{ft}^{2}$. A narrow bearing bar at the center of each leg transmits the leg load to the footing pad, assuring concentric loading of the pad.
A work platform at the top of the crusher thirty-five feet above the ground is of standard construction. Floor plate, floor beams, monorails, columns and bracing support a rock hammer, rock backstops and floor live loads. The columns are supported by the upper deck of the toroid. Monorails serve to access equipment hatches in the upper deck.

A control tower, reaching approximately 60 ft above ground, is supported over one of the three legs and houses ventilation equipment.

### 1.2 Computer Model (Fig. 1)

The basic element used to model the crusher support structure is a quadrilateral isoparametric membrane element which resists inplane forces only. Out of plane bending stiffnesses may be neglected because of the semimonocoque nature of the structure. The maximum element size used is about five feet square. This results in an adequate mesh size for this structural system as the stress and strain distribution in the membrane element is linear.

The computer model was generated in thirds, checking each third with computer plots. The three parts were then combined and duplicate nodes and elements were eliminated at the interfaces. Hatches in the upper deck of the toroid were added. Openings in the exterior panels and openings for doorways were not included in the model. The openings not included have only a small effect on the structure as a whole and their inclusion would only increase the complexity and the computer run time. These openings were analyzed at the time of detailed design.


Fig. 1: Crusher support structure resting on the ground undeformed shape

The upper work platform and control tower completes the computer model. The work platform was modeled using membrane plate elements, beams and bars. Major axis rotations of floor beams were released to simulate a pinned condition at the ends of the beams. The control tower was modeled as a stiff beam with rigid elements connecting it to other elements of the structure.

The crusher was modeled using very stiff beams in a pyramid fashion. The top of the pyramid is located at the crusher center of gravity. The crusher weight is assumed concentrated at the top of the pyramid and is spread out by the stiff beam elements to the series of gusset plates around the inner ring of the toroid.

### 1.3 Static Analysis

Five static loading conditions were investigated:

1) Dead Load
2) Dead Load plus Live Load
3) Dead Load plus Live Load plus $10 \%$ of the Dead Load in the lateral negative Y direction.
4) Dead Load plus Live Load plus $10 \%$ of the Dead Load in the lateral positive $X$ direction

## 5) Thermal Load

The Dead Load consisted of the structure weight, automatically calculated and applied by the computer, plus equipment weights. The Live Load consisted of floor live loads. The lateral loading conditions were provided to estimate the effects of possible blasting of ore in the immediate area of the crusher structure. Also, dynamic loads can be equated to a "pseudo-static" loading condition for evaluation of stresses and fatigue. The thermal load is a change in temperature over one third of the structure and immediately around one leg. The thermal loading condition will approximate the magnitude of stresses due to the unequal heating that might be caused by the sun shining on one side of the structure.
Three different sets of boundary conditions were used in the static analysis. The first set was used for vertical loading conditions only, with the structure resting on the ground. All three legs were assumed to be supported vertically at the center of the leg base. Two of the legs were free to move laterally as though on rollers while the third leg was restrained laterally in two directions to create a stable condition.

The purpose of supporting the legs on rollers was to assume a conservative set of boundary conditions for the vertical load case. If all of the legs were restrained against translation, the lateral component of the reaction required to prevent the translation would exceed any friction that could be developed either between the leg and the footing pad or between the footing pad and the soil. The legs, therefore, were allowed to displace laterally. This displacement increases bending stresses in the legs and is therefore conservative.
The second set of boundary conditions supported all of the legs vertically and laterally at the center of the leg base. This set was used only to determine element stresses due to lateral operating loads while the structure is resting on the ground.
The reason these two sets of boundary conditions are justifiable is due to the sequential nature of the loading.

When the structure is in place and under vertical load, the legs will exhibit an initial radial displacement because the
frictional restraint is inadequate to overcome the lateral reaction required to prevent the displacement. Once relieved, however, the legs can be expected to develop sufficient frictional resistance to restrain a lateral loading of at least ten percent of the vertical dead load reaction. This is the only means by which the structure can resist lateral loads.
The third set of boundary conditions represented the case when the structure is supported on the transporter. The structure was assumed to be supported vertically and laterally at the bottom of the inner ring of the torus directly below the series of gusset plates that support the crusher. The structure weight is also supported by the gussets. The load path of the structure's weight during transport is just reverse of the load path of the crusher weight when resting on the ground.
Summaries of selected element stresses for the three boundary conditions and various loading conditions are shown in Table 1.


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** Legs are restrained from moving laterally.
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Table 1: Maximum element stress - operating condition

### 1.4 Elastic Stability

## Exterior Panels

Exterior panels close off the interior of the toroid and form the fifteen sides of the basic structure. These panels act as deep beams to transfer the upper and lower deck loads to the legs. A relatively thin plate is sufficient to ensure low stresses and is stiffened to prevent buckling.
A stability analysis of the exterior panels was performed on the computer using plate elements. Vertical channel stiffeners were used at the third points of a typical 123 inch square panel which was modeled with 36 elements. Out of plane
displacements were restrained at the edges of the plate. Buckling stresses for four loading cases were obtained.

The load cases were for uniform compression in the horizontal and vertical directions (two cases), pure shear and uniform vertical bending. The results are shown in Table 2.

| Loading Case | Cricical <br> Stress <br> fripsi | Allowable <br> Stress <br> for psi |
| :--- | :---: | :---: |
| Uniform Compression <br> Perpendicular To Stiffeners <br> Uniform Compression <br> Parallel To Stiffeners <br> Uniform Shear <br> Uniform Bending | 2,218 | 1,109 |

Table 2: Buckling stress of stiffened exterior panels

The largest shear stresses in the exterior panels occur near the support legs. The maximum shear stress ( $f_{s}$ ) was found to be approximately $1,200 \mathrm{psi}$. The accompanying vertical bending stress $\left(f_{b}\right)$ was approximately 11,000 psi. The uniform compression ( $f_{c}$ ) in the principle directions was negligible.

The interaction formula for stresses due to bending and shear is given by:

$$
\frac{f_{\mathrm{c}}}{f_{\mathrm{cr}}}+\left(\frac{f_{\mathrm{b}}}{f_{\mathrm{crb}}}\right)^{2}+\left(\frac{f_{\mathrm{s}}}{f_{\mathrm{crs}}}\right)^{2} \leq 1.0
$$

For the stresses given above and allowable stresses given in Table 2:

$$
0+\left(\frac{11,000}{12,762}\right)^{2}+\left(\frac{1,200}{3,094}\right)^{2}=0.89
$$

The result of the interaction formula is less than 1.0, so the exterior panel is within the allowable stresses for this loading condition.
The largest bending stress in the exterior panels occurs at mid-span between support legs. However, this maximum stress is less than half of the allowable bending stress based on buckling ( $12,762 \mathrm{psi}$ ) and is therefore safe.
The stiffening pattern used provides a large increase in strength over the unstiffened plate without adding much weight to the structure.

## Leg Shear Walls

The interior leg shear walls are very important structural elements. These shear walls transfer most of the load on the structure down into the support legs. It is necessary for the walls to have doonways in them to allow access to rooms inside the toroid. Prevention of buckling of the walls in shear is a very important aspect of the design.

A stability analysis was performed by the computer for one wall using plate elements. The doorway in the wall was stiffened all around by use of structural tubes that provide a torsional box. The finite element model was made up of 102
plate elements. Out-of-plane displacements were restrained at the edges of the plate as well as along a vertical line supported by interior walls that stiffen the shear wall.
The critical buckling stress in shear was found from the computer analysis to be $65,460 \mathrm{psi}$. The allowable shear stress considering elastic buckling is then 32,730 psi. The allowable stress considering buckling is larger than the allowable stress in shear considering strength, which is 14,400 psi. Therefore, buckling is not critical for the leg shear walls.

### 1.5 Dynamic Analysis

The static and buckling analysis established plate thicknesses and stiffening patterns based on static loads due to structure and equipment dead loads, live loads, lateral loads and thermal loads. Element stresses must also be checked for the effects of the dynamic load of the crusher. Vibrational amplitudes caused by the crusher dynamic load are important and must be investigated to ensure that excessive vibration does not occur.
The crusher to be installed in the portable structure is an Allis-Chalmers $60 / 89$ gyratory crusher which weighs approximately 450 tons. Crushing is done by the mantle which swings in a circular motion at an angular velocity of 126 cycles per minute (CPM). The mantle rests in the eccentric at the base of the crusher and crushes the ore against the outer shell. The mantle of the crusher weighs 146,000 pounds.
The crusher dynamic load is the force that is required to keep the mantle rotating eccentrically instead of flying out due to the centripetal acceleration. The dynamic force $(\mathbb{P})$ is given by:

$$
P=M W^{2} r
$$

Where $M$ is the mantle mass, $W$ the angular velocity in radians per second and $r$ the eccentricity of the mantle at its center of gravity. The eccentricity, $r$, for a mantle throw of 1 . $3 / 8$ inches is 0.58 inch. The dynamic force is then:

$$
\begin{aligned}
P & =\frac{146,000 \mathrm{lbs} .}{32.2 \times 12} 126^{2} \mathrm{CPM}\left(\frac{2 \pi}{60}\right)^{2}(0.58) \\
& =38,154 \text { pounds. }
\end{aligned}
$$

The dynamic load acts at the center of gravity of the mantle and rotates in a horizontal plane. This load was input as two sinusoidal forcing functions ninety degrees out of phase. The load was applied at the center of gravity of the mantle at 126 CPM with the two forcing functions simulating a horizontal rotating load.
The program used a subspace interaction method to calculate five frequencies and mode shapes of the structure. No degrees of freedom were suppressed due to the complexity of the structure. The boundary conditions used for the dynamic analysis were the same as for the static lateral load analysis, that is, all three legs were restrained from motion in three directions. The first five natural frequencies of the structure are listed in Table 3.
The fundamental frequency was found to be about 2.5 times the crusher frequency of 126 CPM. A frequency ratio of at least 2.0 is desirable to prevent occurrence of sympathetic vibrations with the resulting large member stresses and structure displacements.

| Frequency Number | Eigenvalue Frequency Extraction from Computer Analysis |  | Hand Calculated Frequency From Single Degree of Freedom System |  | Percent Difference |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 321 | 0.187 | 307 | 0.195 | 4.6 |
| 2 | 331 | 0.181 | 324 | 0.185 | 2.2 |
| 3 | 388 | 0.155 | 400 | 0.150 | 3.0 |
| 4 | 455 | 0.132 | - | - | - |
| 5 | 462 | 0.130 | 442 | 0.136 | 4.5 |

Table 3: Natural frequencies

The first two frequencies were primarily a vibration in the $Y$ coordinate and X -coordinate directions respectively (Fig.2). The third frequency was associated with the X-direction swaying of the control tower above the toroid. The fourth frequency was a torsional mode about the vertical $(Z)$ axis while the fifth mode was associated with a vertical $(Z)$ vibration. These frequencies were verified approximately by hand by assuming a simplified single degree of freedom system utilizing the stiffness calculated from the static computer results (i.e., $w^{2}=k / m$ ). Close agreement was obtained as shown in Table 3 which gives confidence in the convergence of the computer solution.


Fig. 2: Crusher support structure resting on the ground deformed shape of the fundamental mode of vibration

The computer program used the five natural frequencies and the dynamic load to calculate selected member stresses and vibrational amplitudes. The time period considered covered five seconds, or about ten revolutions of the crusher mantle. Structural damping was assumed to be five percent of critical damping. The five second period considered was sufficient to produce steady state conditions.
The result of the forced frequency analysis showed member stresses due to the crusher dynamic load to be quite low. The dynamic stresses were approximately equivalent to a static lateral load (See Table 4) on the structure of three percent of the structure dead load. Comparing this to the static load case of a ten percent lateral structure dead load shows that plate thicknesses established by static load cases are adequate for the dynamic load case.

| Element | Stress <br> (psi) |
| :--- | :---: |
| Upper Deck | 318 |
| Lower Deck | 198 |
| Leg Plates | 150 |
| Leg Shear Walls | 249 |
| Exterior Panels | 187 |
| Interior Ring | 217 |
| Interior Gussets | 482 |
| Crusher Support Ledge | 262 |

Table 4: Maximum element stresses due to the dynamic load

Vibrational amplitudes were investigated by using the dynamic load and superimposing the five mode shapes. Amplitudes of vibration were obtained for five percent critical damping. The maximum half-amplitude of vibration at the crusher was calculated to be 10 mls .

A second forced response analysis was run to determine what influence the structural damping had on the vibrational amplitudes. A value of 2-1/2 percent of critical damping was used in the second analysis. The resulting vibrational amplitudes did not increase significantly. The amplitudes of vibration are, therefore, not very sensitive to the structural damping assumed in the damping range considered.

Individual structural elements were analyzed on a member by member basis to prevent sympathetic vibrations; beams and exterior panels were designed for natural frequencies greater than 400 CPM.

### 1.6 Technical Summary and Conclusion

A finite element structural analysis was performed to determine the feasibility of a portable crusher structure. The finite element model consisted of 625 nodes and approximately 1,800 elements. The program solved a system of 3,750 simultaneous equations to obtain solutions.
Static loading cases were evaluated to determine member stresses. Static loading cases included dead load, live load, lateral load and thermal load. Static member stresses were checked against allowable stresses both for strength and for elastic buckling. Plate thicknesses and stiffening patterns were established.
Natural frequencies were calculated to evaluate the effects of the crusher dynamic load. The fundamental frequency of the structure was found to be 321 CPM which is 2.5 times the crusher frequency. Stresses and amplitudes due to the dynamic load were found to be small.
The overall structural system of the portable structure is quite stiff. Member stresses are well within allowable stresses for both strength and buckling. Vibrational amplitudes are small and appear to be tolerable. The weight of the structure is approximately 180 tons.

## 2. Space Frame Structure

### 2.1 Geometry

The basic structure used to support the crusher is a four legged space frame. Fig. 3 shows a model of the overall crusher system. This system employs a movable apron feeder which rides over the crusher structure during the transport mode and which also supports the mantle crane during the service mode. Fig. 4 shows the system during the transport mode. The main structure corresponds to the lower platform level.

The four square legs are 20 ft on center by 40 ft on center. Approximately 12 ft of clearance exists between the main support girders and the ground level. The four main girders are 4 ft by $4-1 / 2 \mathrm{ft}$. The overall dimensions of the crusher support framing plan including service walkways are 40 ft by 46 ft .
In addition to supporting the crusher, this structure supports the remaining superstructure.

### 2.2 Computer Model (Fig. 5)

Fig. 5 shows a simplified computer generated finite element model, mainly using quadrilateral, isoparametric membrane elements. The structure was modeled with 420 nodes and 730 elements, that resulted in 2,520 simultaneous equations. Similiar to the plate shell structure, the legs were assumed free to slide for vertical static loading, were assumed pinned against movement for dynamic analysis, and were assumed free when the structure is supported by the transporter. In addition, another boundary condition was considered for this structure; namely, the condition of the structure being solely supported by two diagonally opposite legs. This method of support would correspond to possible differential settlement of one of the legs which could result in the structure rocking on two of its other legs.

### 2.3 Static Analysis

Five static loading conditions were investigated:

1. Dead Load
2. Dead Load plus Live Load
3. Dead Load plus Live Load plus $10 \%$ of Dead Load in lateral X-direction.
4. Dead Load plus Live Load plus $10 \%$ Dead Load in lateral Y-direction.
5. Dead Load plus Apron Feeder Load during servicing and transport modes.
These loading conditions were completed with the appro priate boundary condition to generate a definitive structural analysis of the system.
Stress levels were kept low in the main support member to reduce the potential for fatigue failures due to vibration. Maximum stresses for the operating mode (dead load plus live load) were 2,970 psi for the leg columns and 14,300 psi for the longitudinal girders.
Maximum stresses for the service mode (dead load plus live load plus apron feeder) were 6,900 psi for the leg columns and 17,700 psi for the longitudinal girders.
Maximum stresses for the transport mode were considerably lower since the load path for the crusher is almost directly through the transporter to the ground.


Fig. 3: Model of overall crusher system


Fig. 4: Crusher system during transport mode


Fig. 5: MSME movable crusher

### 2.4 Elastic Stability

Because of the more compact nature of this design, detailed elastic stability analysis was not required. The natural configuration is such that stiffeners were not required to achieve allowable code values.

### 2.5 Dynamic Analysis

Vibrational amplitudes caused by the crusher dynamic load were investigated by a slightly different method than that used for the plate shell structure. This computer analysis utilized the Rayleigh-Ritz technique with consideration given to mass concentrations at each of the 420 nodes. The formula used to calculate the natural frequencies is:

$$
w^{2}=g \frac{\sum m_{\mathrm{i}} d_{\mathrm{i}}}{\sum m_{\mathrm{i}} d_{\mathrm{i}}^{2}}
$$

where
$m_{\mathrm{i}}=$ concentrated mass at node i
$d_{\mathrm{i}}=$ deflection at node i corresponding to assumed mode shape
$g=$ acceleration due to gravity
$w=$ frequency

Lowest natural frequencies resulting from this analysis were:
730 cpm in lateral X-direction
580 cpm in lateral Y-direction
640 cpm in vertical Z-direction
Torsional fundamental modes were higher in frequency.
Stress runs were also carried out for the load cases approximating crusher rotation. These lateral loads due to crusher rotation were approximately $4 \%$ of vertical dead loading and the resulting stresses were not critical. The structure weighs approximately 160 tons.
The maximum half-amplitude of vibration at the crusher base was calculated to be six mils.

### 2.6 Comparison of Structures

The space frame structure as designed houses a 54-74 gyratory crusher; whereas, the plate shell structure houses a 60-89 gyratory crusher. The plate shell structure has the advantage of using only three legs which establishes a definite loading pattern. The space frame is a more rigid (i.e., higher natural frequency) structure, but runs the risk of rocking on two of its opposite legs should significant differential settlement occur. Stress levels and deflections were comparable for both structures.
Both structures certainly deserved a finite element analysis which is fairly routine in this technological era. Such analysis avoids unwarranted conservatism in static analysis and more accurately and safely predicts dynamic response.

