Dedicated to Professor Dr.-Ing. E. Bahke on his 65th birthday

Consideration of the Effect of a Wide Particle Size Distribution on Calculations for Hydraulic Conveying

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Summary

The use of empirical and semi-empirical equations for the analysis of flow behaviour in the hydraulic transport of solids is long established. These equations, however, do not apply in the case of wide particle size distributions. The author extends the use of one such equation from Durand to this case and discusses the influence of size and density distributions on such analyses.

Nomenclature

b_d	ratio of size distribution	
b_w	ratio of settling velocity distribution	
c_T	transport concentration by volume	
c_V	in situ concentration by volume	
c_w	drag coefficient	
D	pipe diameter	m
$d_{90\%}$	particle size at 90 % passing	m
$d_{10\%}$	particle size at 10 % passing	m
d_s	particle size	m
d_{sm}	mean particle size	m
g	gravitational acceleration	m/s²
K	constant	
L	length	m
m	exponent	
n	exponent	
x_i	component	
$\Delta p/L$	total pressure gradient	N/m³
$(\Delta p/L)_w$	pressure gradient of water	N/m³
v_m	velocity of mixture	m/s
$w_{s90\%}$	settling velocity at 90 % passing	m/s
$w_{s10\%}$	settling velocity at 10 % passing	m/s
ϱ_s	density of solids	kg/m³
ϱ_w	density of water	kg/m³
ϕ	expression in Eqs. 3 and 5	
ψ	expression in Eqs. 3 and 5	

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1. Introduction

If a plant for horizontal hydraulic transport of solids is to be designed, the expected pressure loss must be known as exactly as possible because of its decisive influence on such values as pipe diameter, installed power and transport velocity. As the expenses for design tests in most cases are too high, empirical and semi-empirical equations have been developed, so as to be able to predict theoretically the pressure loss for solids conveying. Besides plant data such as pipe diameter, velocity, pressure loss and pipe length, many more parameters are determined by the solids themselves and these usually vary from one solid to the other. Among these are density, average particle diameter, particle shape, friction behaviour and particle-size distribution. Tests to determine the influence of the particle-size distribution have been performed within a research project funded by the German Research Foundation (DFG).

2. Characterization of a Particle-Size Distribution

Normally, a bulk solid is characterized by determining its density and its average particle diameter by means of a sieve analysis. In this case by "average particle diameter" the average particle size is understood, i.e., the arithmetic average of the particle sizes, taking into account their frequency. The particle-size distribution can easily be shown graphically by plotting the sieve sizes against the accumulated remainder [1].

In order to evaluate numerically the particle-size distribution two values are defined. From the curve of the percentages passed two particle sizes result, one for 90 % passed, one for 10 % passed, the quotient of which defines the particle-distribution quotient

$$b_d = \frac{d_{90\%}}{d_{10\%}} \quad (1)$$

The particle size, however, defines only a geometric value in which not much is said about the flow behaviour. For this purpose, the settling velocity of the particles is more appropriate because it shows the varying flow behaviour of the fractions of a conveyed material, similar to a curve of percentage passed if it is plotted sieve-fraction-wise.

Analogously to the above, a quotient of the settling velocity distribution can be defined as

$$b_w = \frac{w_{s\,90\,\%}}{w_{s\,10\,\%}} \qquad (2)$$

This quotient is a measure of the particle-size distribution as well, because of its definition however, it is also valid for solids with components of different densities, as for instance in ore mixtures.

3. Description of the Materials Conveyed

In the experimental investigations of the influence of the particle-size distribution only heterogeneous mixtures were used, i.e., mixtures which show a distinct concentration profile in transport. In order to eliminate the adulterating influence of different particle shapes only spherical solids were chosen. Two different densities (glass and steel) with three uniform fractions were at hand, from which particle-size mixtures were produced as desired. Fig. 1 shows the sieve analyses of the solids consisting of glass spheres which contain uniform solids as well as the two- and three-fraction mixtures. Fig. 2 shows the corresponding curves of

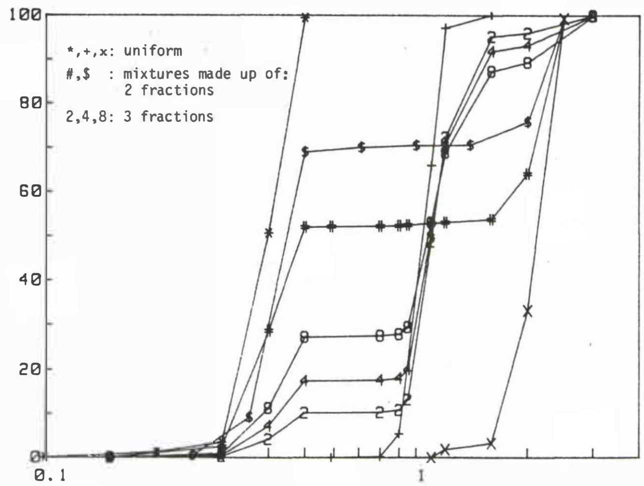

Fig. 1: Sieve analysis for tested solids (glass spheres)

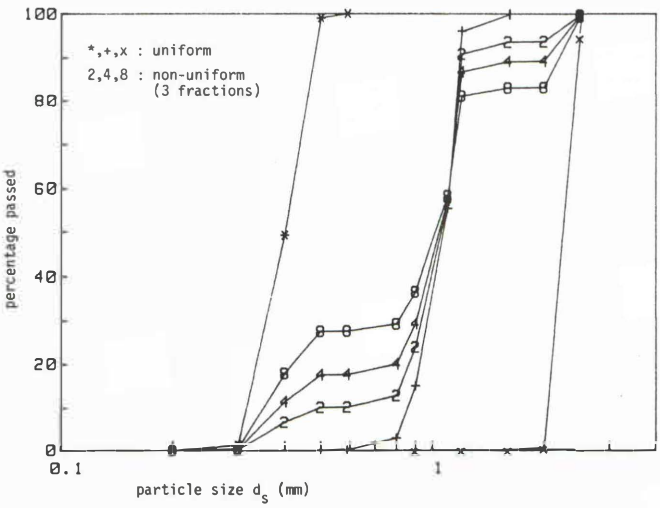

Fig. 2: Sieve analysis for tested solids (steel spheres)

percentage passed for steel spheres, the characteristic value b_d indicating the width of distribution ranges from 1.2 for uniform solid up to 6.9 for mixtures.

Besides the solids shown in the sieve analyses, additional mixtures were produced, among others mixtures consisting of glass and steel spheres. To characterize them, however, the sieve curve with percentage passed expressed in volume percent was no longer sufficient. In addition, the settling velocity distribution had to be determined.

4. Description of the Transport Tests

The tests with uniform and non-uniform solids were performed in a transport test loop with a nominal diameter of 80 mm, which is shown in Fig. 3 and has already been used

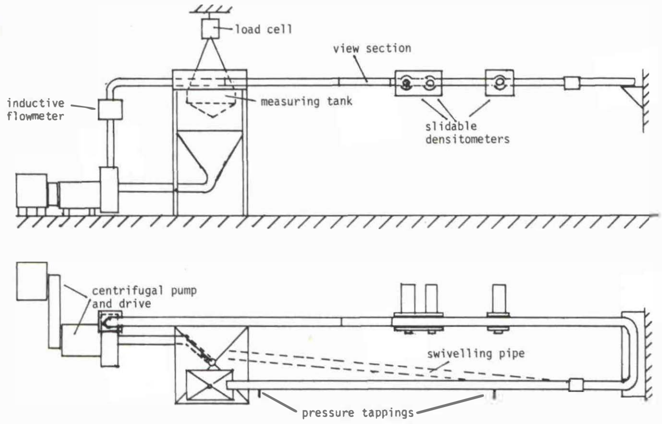

Fig. 3: Schematic view of the test loop

by Gödde [2]. The plant will be described once more, however, because some modifications were made. The test loop was driven by a centrifugal pump with hardened lining and a speed-controlled motor. Along the conveying pipe, transducers for the determination of the flow condition were installed. An electromagnetic flowmeter was used to measure the velocity of the mixture. This was fastened in the vertical part of the pipe in order to obtain an axially symmetrical flow profile in the transducer. In the horizontal pipe three vertically adjustable radiometric density meters were installed. One of them was equipped with a Cs 137-source and served to determine the density profile. The other two devices were fixed relative to each other, but mounted on a slideable frame. The associated amplifiers had low integration intervals so that, after the passage through band filters, the density fluctuations were available. A connected correlation computer compared the flow signals and shifted them against each other until they showed maximum similarity. This cross-correlation method allowed the determination of the transit time of the density signal between the two transducers, from which resulted the solids velocity. In the conveying pipe there were two pressure-tappings to which a pressure-difference transducer for measuring the pressure-gradient was connected. By means of the slewable part of the return pipe the discharge could be measured and thus it was possible to measure the mixture velocity from filling-volume and filling-time and the transport concentration from mass flow and volume flow.

The connection to a digital computer for determining and processing the measured values led to significant modifications of the test rig compared to its previous operation. This device rapidly scanned the output signals of the amplifiers and stored them for subsequent evaluation. Thus another method for the determination of the transit time could be applied by digitizing the density fluctuations with 1,000 Hz and then calculating the cross-correlation. The test performance could be accelerated to a very large degree by means of this system so that the attrition of the solids was considerably diminished and the stability of the solid characteristics in transport was assured. Though a sieve analysis was performed before and after each test series, hardly any attrition of the solids was noticed.

In each solid three or four concentrations were tested. For each curve several profile and discharge measurements were run, while pressure-gradient and inductive velocity were determined at each point. In each test the transport velocity was varied from maximum power of the driving-motor to sub-critical velocity, so that the settling behaviour could also be observed.

5. Test Results

The pressure-loss results were plotted against the average transport velocity and show a course typical for heterogeneous mixtures. For high velocities, the pressure-loss curve approaches the curve for clear water, but for low velocities deviates from it and shows a more or less distinct minimum. If, however, a uniform solid with a particle size distribution of $b_d = 1.2$ is compared to non-uniform mixtures with equal density (with b_d from 2 to 5), different curves with regard to inclination and absolute height are obtained, as shown in Fig. 4, though the solids depicted have equal average particle diameters.

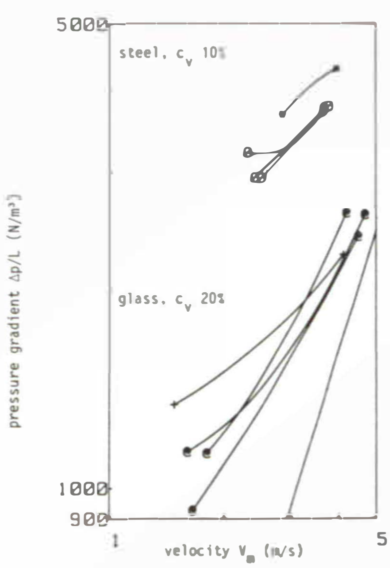

Fig. 4: Pressure loss curves for steel spheres and glass spheres with $d_{sm} \approx 1$ mm

•, + uniform

0, @ non-uniform (3 fractions)

In this case, the mixtures were composed of 3 fractions, the main part of which consisted of the 1 mm fraction shown for comparison. In Fig. 5, however, the uniform fraction with $d_s = 1$ mm is compared to two mixtures consisting of 0.4 mm and 2 mm fractions, the average particle size of which is also about 1 mm (0.92 and 1.2 mm).

Here, the discrepancy of the curves is even more obvious. It is caused by the width of the particle-size distribution and therefore prevents one applying the equations for uniform solids to non-uniform ones. These results are confirmed in [3] and [4].

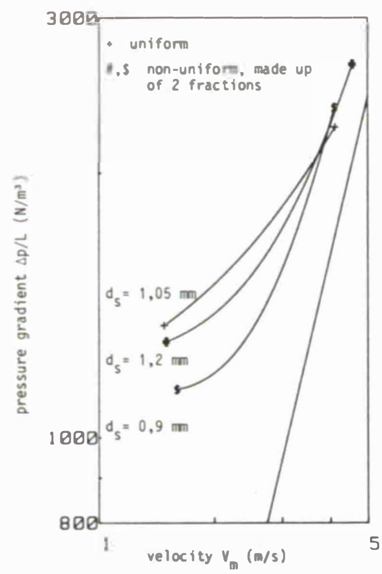

Fig. 5: Pressure loss curves for glass spheres at $c_v = 20\,\%$

6. Evaluation According to Durand's Equation

Durand developed an equation which yields good accordance with test results from uniform solids:

$$\phi = K \cdot \psi^n \qquad (3)$$

where

$$\phi = \frac{\frac{\Delta p}{L} - \left(\frac{\Delta p}{L}\right)_w}{\left(\frac{\Delta p}{L}\right)_w \cdot c_T}$$

$$\psi = \frac{gD(\varrho_s - \varrho_w)}{v_m^2 \sqrt{c_w}\,\varrho_w}$$

K — coefficient
n — exponent

The free parameters of the equation were determined as $K = 83$ and $n = 1.5$. These parameters result as coefficients of a straight line equation if the left and right hand sides of the equation are plotted on double-logarithmic paper.

The test results obtained for uniform solids confirm the validity of the equation, the coefficients of the straight line from a regression analysis being determined as $K = 84.8$ and $n = 1.46$. Besides these measured values, the results for non-uniform solids are also plotted in Fig. 6, whose equalizing straight line shows a lower and flatter course, so that the coefficients amount to only $K = 72$ and 1.28, thus confirming the differences already noticed in the pressure-loss diagrams.

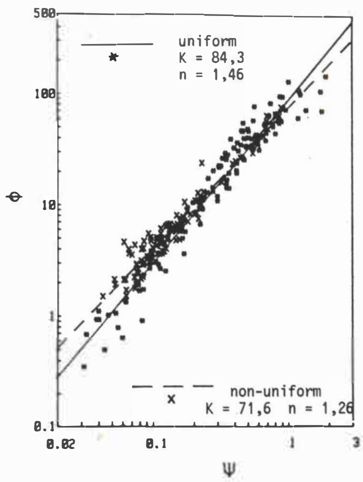

Fig. 6: Measured points and regression curves, plotted according to Eq. 3

In Eq. 3 the characteristics of the solids are taken into account by the density and drag coefficient of the average particle size. In order to make the equation applicable for non-uniform solids, Condolios and Chapus [6] proposed the use of an average drag coefficient with

$$\sqrt{c_W} = \Sigma x_i \sqrt{c_{Wi}} \quad (4)$$

where

c_W = drag coefficient of the particle size mixture
c_{Wi} = drag coefficient of a sieve fraction
x_i = component of the i^{th} fraction

7. Extension of Durand's Equation

In order to account for the width of a particle size distribution and thus apply Eq. 3 to non-uniform solids, another exponent m was introduced, so that the equation is now

$$\phi^m = K \cdot \psi^{n/m^2} \quad (5a)$$

or transformed

$$\phi = K^{1/m} \cdot \psi^{n/m^3} \quad (5b)$$

where $m = 2 - \left(\frac{d_{s\,90\,\%}}{d_{s\,10\,\%}}\right)^{-0.04} = 2 - b_d^{-0.04}$

The exponent m should meet the requirement that for uniform solids $m = 1$, i.e., that the original form of the equation with its coefficients is maintained.

Both versions of the equation, 5a und 5b are obtained by simple algebraic transformation. Version 5a is appropriate for the determination of the coefficients K and m for a particular solid from test results, while version 5b serves for the calculation of pressure losses for a known solid.

Fig. 7 shows the plot of the values measured according to the extended equation, version 5a. Although the same values measured as in Fig. 6 were used, the equalizing straight lines of uniform and non-uniform solids coincide. Eq. 5 thus seems to reproduce very closely the influence of a wide particle size distribution on the pressure-loss curve. The differences between the original coefficients K and n of Eq. 3 result from the fact that the exponent m for uniform solids is a little above unity.

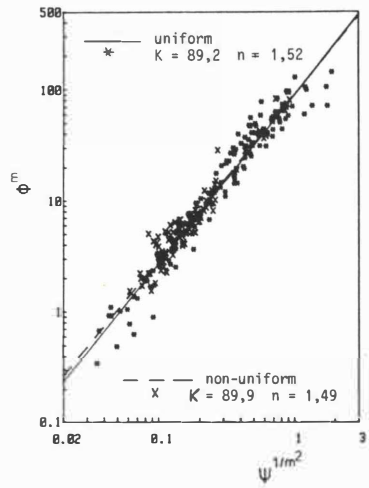

Fig. 7: Measured points and regression curves, plotted according to Eq. 5

In addition to the test results with steel and glass spheres, not all of which are depicted in pressure-loss diagrams, mixtures of these two test materials were investigated in the test loop. Because of the different densities of the two components a characterization of the particle-size distribution with the characteristic value b_d, as described above, does not here make sense. Instead, the ratio of the settling velocity b_w should be used. The evaluation of the results with this characteristic value, however, has not yet been finalised.

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