# Correlation Analyses in the Design of Pneumatic Transport Plant 

Manfred Weber, Germany

## Summary

Correlation analyses have long been applied to hydraulic transport with considerable success. The author reviews the possibilities in using such an approach with pneumatic conveying. The lack of suitable experimental data is a real problem but using the results available some promising correlations are achieved.

## Nomenclature

$C_{T} \quad$ transport concentration, $\dot{V}_{s} I \dot{V}_{\text {tot }}$
$C_{\text {W }}$ drag coefficient of a sphere
$d_{s} \quad$ particle diameter
$D$ pipe diameter
Fr Froude number (related to pipe), $v^{2} f / D g$
$\mathrm{Fr}_{\mathrm{s}} \quad$ Froude number (related to solids), $w_{\text {sol }}^{2} / d_{s} g$
8 gravitational acceleration
$K$ coefficient in Durand's equation
$L$ length
$\dot{M}$ mass flow rate
$n$ exponent in Durand's equation
$p$ pressure
$v$ velocity
$\checkmark$ volume flow rate
$\lambda$ pipe friction coefficient
$\mu \quad$ mixing ratio, $\dot{M}_{s} / \dot{M}_{f}$
@ density

## Subscripts

f fluid (gas or liquid)
$m$ mixture
s solid
tot total

- related to area of pipe

[^0]
## 1. Introduction

The transport of solids in flowing carrier mediums can be calculated analytically only to a certain limit because of the numerous influences of the materials involved, especially in dispersed solids. This is why in this field empiricism has much more importance than in clear fluid flow. Therefore, the calculation and the design of pneumatic and hydraulic transport plants are based mainly on empirical and semi-empirical analyses.

## 2. Comparison of Pneumatic and Hydraulic Transport

For basic reasons, the material-specific influences of the solids to be transported have many more consequences on pneumatic than on hydraulic transport, as already described in previous publications. This can also be seen very clearly in the equations for calculation that are offered in the literature. While for calculating the pressure loss in hydraulic transport mainly Durand's equation or slightly modified versions of it are used

$$
\left.\left.\begin{array}{c}
\Delta p=\Delta p_{\mathrm{f}}+\Delta p_{\mathrm{s}} \\
=\left[\lambda_{\mathrm{f}}+\lambda_{\mathrm{f}} C_{\mathrm{T}} K\left(\frac{g D}{v_{\mathrm{m}}^{2}}\right.\right. \tag{1}
\end{array} \frac{\mathrm{e}_{\mathrm{s}}-\mathrm{e}_{\mathrm{f}}}{\varrho_{\mathrm{f}} \sqrt{ } C_{\mathrm{w}}}\right)^{n}\right] \frac{\mathrm{e}_{\mathrm{f}}}{2} v_{\mathrm{m}}^{2} \frac{L}{D}
$$

in the literature on pneumatic transport quite often the statement

$$
\begin{equation*}
\Delta p=\Delta p_{\mathrm{f}}+\Delta p_{\mathrm{s}}=\left(\lambda_{\mathrm{f}}+\mu \lambda_{\mathrm{s}}\right) \frac{e_{\mathrm{f}}}{2} v_{\mathrm{fo}}^{2} \frac{L}{D} \tag{2}
\end{equation*}
$$

is found.
When comparing both equations it becomes obvious that in hydraulic transport a more general formulation is used, applying characteristic numbers derived from the laws of similitude. These numbers can be adjusted specifically to the conveyed material and even more precisely with regard to particle distribution and particle shape by means of the coefficient $K$ and the exponent $n$. For uniform and spherical solids Eq. (1) with $K=83$ and $n=1.5$ can be taken as generally valid.

For pneumatic transport one notices that Eq. (2) contains only the mixture ratio as a characteristic number of the mechanics of similitude of the mixture. All the other important influences, such as particle size, solids density, pipe diameter, air velocity, air density, impact number, coefficient of mechanical friction etc., are contained in the additional pressure-loss coefficient of the solid $\lambda_{\mathrm{s}}$, which is determined separately for each material.
Thus it is true that Eq. (2) is applicable for different conditions of pneumatic transport if the appropriate pressure-loss coefficient is used, but a complete representation such as in Eq. (1) for hydraulic transport is not possible.
If a more generally valid representation for pneumatic transport is desired, measured data of solids of different densities, particle sizes, pipe diameters, mixture ratios and transport velocities must be summarized by means of an appropriate correlation with a standard deviation as low as possible. To set up suitable power products characteristic values of similitude of the gas-solid-mixtures are used as factors.

## 3. Correlations in Pneumatic Transport

In practical use, however, great difficulties arise. Even if only the field of dilute phase flow and nearly uniform solids are taken into consideration, it is difficult to find enough complete measured data for a correlation. Stegmaier [4] has summarized several fine-granular solids for horizontal transport by a correlation which comprises some of the above mentioned factors in characteristic numbers of similitude and presented them as a power product (Fig. 1). As an average value for the most differing solids he found for the additional pressure-loss coefficient:

$$
\begin{equation*}
\lambda_{\mathrm{s}}=2.1 \cdot \mu^{-0.3} \cdot \mathrm{Fr}^{-1} \cdot \mathrm{Fr}_{\mathrm{s}}^{0.25}\left(D / d_{\mathrm{s}}\right)^{0.1} \tag{3}
\end{equation*}
$$

If the relatively high standard deviation is not acceptable, each type of solids can be correlated separately. For pure clay, for instance, the result is more precisely:

$$
\begin{equation*}
\lambda_{\mathrm{s}}=6.2 \cdot \mu^{-0.3} \cdot \mathrm{Fr}^{-1.2} \cdot \mathrm{Fr}_{\mathrm{s}}^{0.25}\left(D / d_{\mathrm{s}}\right)^{0.1} \tag{4}
\end{equation*}
$$

For the horizontal transport of different coarse-granular and nearly uniform solids the same correlation was performed. As measured data, Siegel's test results [5] for polystyrole, spheres of glass and steel in different pipe diameters were applied. It can be seen that the same power product statement that Stegmaier found for fine-granular solids summarizes coarse-granular solids as well. As can be seen from Fig. 2, the average value for polystyrole of different particle sizes conveyed in pipes of different diameters as well as for steel and glass spheres is considerably lower than for finegranular solids. The ascent of the straight line for the average value, i.e., the dependence on the Froude number, however, is nearly equal. The standard deviation for the regression analysis of the three solids amounts to $42.6 \%$ and is comparatively high. The pressure-loss coefficient for all of the three solids amounts to:

$$
\begin{equation*}
\lambda_{\mathrm{s}}=0.082 \cdot \mu^{-0.3} \cdot \mathrm{Fr}^{-0.86} \cdot \mathrm{Fr}_{\mathrm{s}}^{0.25}\left(D / d_{\mathrm{s}}\right)^{0.1} \tag{5}
\end{equation*}
$$

For polystyrole on its own one finds more precisely, with a standard deviation of $44 \%$ :

$$
\begin{equation*}
\lambda_{\mathrm{s}}=0.041 \cdot \mu^{-0.3} \cdot \mathrm{Fr}^{-0.76} \cdot \mathrm{Fr}_{\mathrm{s}}^{0.25}\left(D / d_{\mathrm{s}}\right)^{0.1} \tag{6}
\end{equation*}
$$



Fig. 1: Pressure loss coefficient for fine particles according to Stegmaier[4]


Fig. 2: Correlation of pressure loss coefficient for horizontal pneumatic conveyance of coarse materials according to data measured by Siegel [5]. The mean value of the 132 points is represented by the dotted line.

For glass spheres the single correlation, with a standard deviation of $21 \%$, yields:

$$
\begin{equation*}
\lambda_{\mathrm{s}}=0.021 \cdot \mu^{-0.3} \cdot \mathrm{Fr}^{-0.65} \cdot \mathrm{Fr}_{\mathrm{s}}{ }^{025}\left(D / d_{\mathrm{s}}\right)^{0.1} \tag{7}
\end{equation*}
$$

For steel spheres, with $18 \%$ standard deviation, the result is:

$$
\begin{equation*}
\lambda_{\mathrm{s}}=0.038 \cdot \mu^{-0.3} \cdot \mathrm{Fr}^{-0.72} \cdot \mathrm{Fr}_{\mathrm{s}}{ }^{0.25}\left(\mathrm{D} / d_{\mathrm{s}}\right)^{0.1} \tag{8}
\end{equation*}
$$

These results enable also the calculation of the pneumatic transport of coarse solids without knowing specific measured data where the margin of safety corresponds to the standard deviation. This, however, can only be expected for nearly uniform and spherical solids. For solids that are nonuniform and deviate much from the spherical shape a rotation of the equalizing straight line similar to that for hydraulic transport must be expected. This, however, could be taken into account by an appropriate particle averaging.

It is known from experience in vertical pneumatic transport that the influence of weight prevails at low velocities, but as the velocity increases, friction gains importance. Therefore, in the calculation of the pressure loss one cannot only rely on the weight of the solids, which could be set up theoretically, but it is also necessary to find an empirical relationship for vertical transport from the measured data. For such a correlation, data measured by Flatow [6] were used for polystyrole, glass and steel spheres.
As shown in Fig. 3, a different correlation, containing the influence of gravity as the predominant component and also a correction for the friction component, yields good results. This relationship yields the average pressure-loss coefficient for all three solids in vertical transport as

$$
\begin{equation*}
\lambda_{\mathrm{s}}=\frac{v_{\mathrm{f}} / v_{\mathrm{s}}}{1,200}+\frac{2 v_{\mathrm{f}} / v_{\mathrm{s}}}{\mathrm{Fr}} \tag{9}
\end{equation*}
$$

with a standard deviation of about $15 \%$.

## 4. Conclusions

Thus correlations for the calculation and the design of pneumatic conveying plants for dilute phase flow are available that cover a wide range of values. Especially for optimizations such equations prove to be very advantageous. This is most important for the handling of long conveying lengths where all the influences change additionally with the pipe length, due to compressibility.

|  | $d_{s}(\mathrm{~mm})$ | $D(\mathrm{~mm})$ | $\mu$ |
| :--- | :---: | :---: | :---: |
| + polystrole | $1 / 2.7$ | $50 / 100 / 200$ | $0,5 / 27$ |
| glass <br> spheres | 1.21 | $50 / 100 / 200$ | $0.5 / 19$ |
| steel <br> spheres | 1.13 | $50 / 100 / 200$ | $0.5 / 12$ |



Fig. 3: Correlation of the pressure loss coefficient for vertical pneumatic conveyance according to data measured by Flatow [6]. 251 points.

## References

[1] Weber, M. et al., "Strömungsfördertechnik", Kraußkopf, Mainz 1974
[2] Weber, M., "Principles of Hydraulic and Pneumatic Conveying in Pipes", bulk solids handling Vol. 1 (1981), pp. 57-63
[3] Durand, K., "Basic relationship of the transport of solids in pipes", Minnesota Int. Hydr. Div. A.S.C.E., pp. 89-103, Sept. 1953
[4] Stegmaier, W., "Zur Berechnung der horizontalen pneumatischen Förderung feinkörniger Stoffe", fördern und heben, Vol. 28 (1978), pp. 363-66
[5] Siegel, W., "Experimentelle Untersuchungen zur pneumatischen Förderung kömiger Stoffe in waagerechten Rohren und Überprüfung der Ähnlichkeitsgesetze", VDIForschungsheft 538, 1970
[6] Flatow, J., "Untersuchungen über die pneumatische Flugförderung in lotrechten Rohrleitungen", VDI-Forschungsheft 555, 1973


[^0]:    Prof. Dr.-Ing. M. Weber, Institut for Fördertechnik, Abt. Stromungsfordertechnik, Universităt Karlsruhe (TH), Hertzstr. 16, D. 7500 Karlsruhe 21, Federal Republic of Germany
    Translation of paper delivered at the Conference on Conveying Technology TRANSMATIC 81, September 30-October 2. 1981, organised by the Department of Conveying Technology (Institut for Fördertechnik). University of Karlsruhe, Fed. Rep. of Germany.

