bulk solids handling Volume 2, Number 1, March 1982

Stressing of Rubber Conveyor Belts and Its Mathematical Treatment

Rainer Alles, Germany

Summary
Mathematical investigations of belt stresses permit important conclusions which may lead to the improvement of conveyor belting and system design.

1. Introduction
Manufacturers and users alike are interested in studying and ascertaining as exactly as possible the stressings to which conveyor belts are subjected. Apart from receiving and transporting loads, one of the most important functions of the conveyor belt is the transmission of tension. Consequently tensile strains are the main stressings which are primarily borne by the tension members made of textile or steel cable plies.

The basic principles used in calculating such strains can be traced back to theoretical essays written by Euler in the 18th century. The well-known Eytelwein formula, originally published as early as 1808, is still applied today in an unaltered form to conveyor belt calculations. However, intensive research work in the conveyor belt sector did not start until this century; this phase culminated in 1942 with the publication of German Standards DIN 22101 — a relatively early standardization of the calculating method adopted for belt conveyors.

As might be expected this standard above all dealt with determining conveying capacities and input power, but calculating tensile stresses of belts proceeded from simple assumptions in keeping with the latest technological developments at that time: neither the numerous influences exerted by the design and profile of belt systems nor the possible operating states occurring when starting and stopping them were closely investigated. The lack of knowledge about how certain design features would affect the system was allowed for by incorporating high safety margins.

Since then countless studies have been devoted to investigating frictional forces, phenomena during starting and stopping, and stresses to which conveyor belts are subjected. The current state of the art was reflected recently in the revised edition of DIN 22101. In the following the effects on calculation and construction of conveyor belts are among other topics to be considered.

Dr.-Ing. Rainer Alles, Continental Gummi-Werke AG, P.O. Box 169, D-3000 Hannover 1, Federal Republic of Germany

2. Sequential Calculation
An important and for the most part new requirement is the ascertainment of local belt tension forces and their extreme values along the belt's tight and slack sides. For a particular application it is no longer sufficient merely to concentrate on T_1 at the drive but all points of the installation must be considered, making due allowance for all possible operating and loading states. It has become necessary to introduce the so-called "sequential calculation".

This calculation entails a section-by-section determining of the belt tension forces with regard to the frictional forces existing in these sections, the input of drive forces and, in special cases, forces occurring due to inertia. Only in this way is it possible to ascertain all belt tension forces significant for layout and design even for belt conveyor installations having differing inclinations or with drives and brakes situated at random locations. It goes without saying that this must be preceded by a relevant calculation of the necessary drive power (Fig. 1).

However, since a sequential calulation can refer only to one particular state of operation and loading, several sequential calculations must always be carried out for various operating states actually encountered and coordinated with one another by making appropriate adjustments. It is above all the following limiting factors which have to be met and which consequently influence the adjustments that have to be made:

— Limitation of slip on driven and/or braked pulleys
— Limitation of belt sag at the point along the installation
— Constant total stretch of belt with fixed take-up pulley
— Constant belt tension force at all points of the installation with gravity type take-up pulley
— Limitation of the stopping time or of the stopping distance

Particularly in the case of installations incorporating differently inclined sections there is a multitude of possible operating states which in the first instance require specific investigation and adjustment merely with respect to slip and belt sag. Subsequently these operating states have to be examined and considered in view of further limiting factors in the critical case, and appropriately adjusted to them. The large number of calculations needed in this connection can be illustrated with the aid of an example (Fig. 2).

The discharge belt of a combined spreader with a deflecting boom already has four possible operating states in one

Course of belt tension determination by sequential calculation

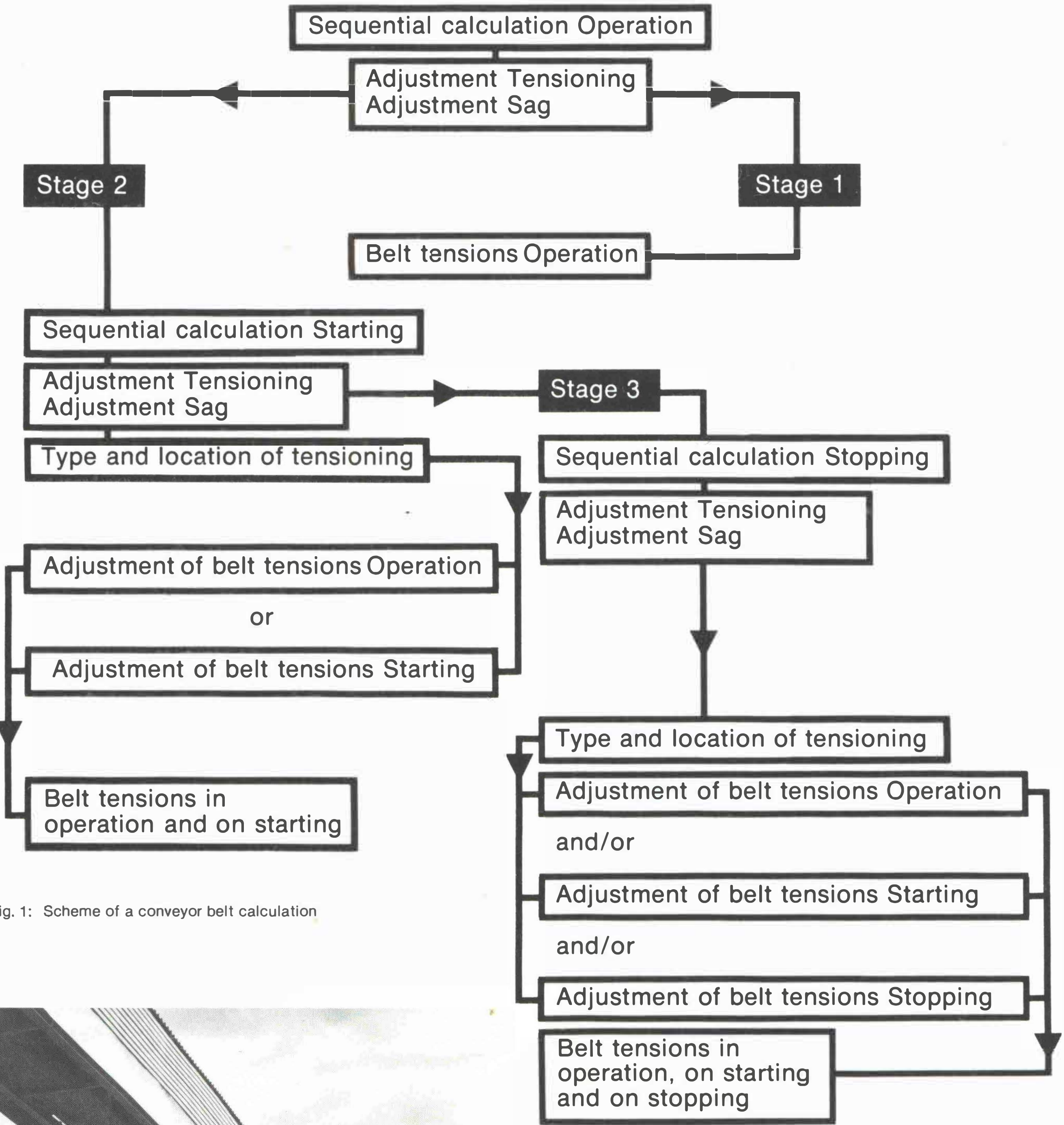

Fig. 1: Scheme of a conveyor belt calculation

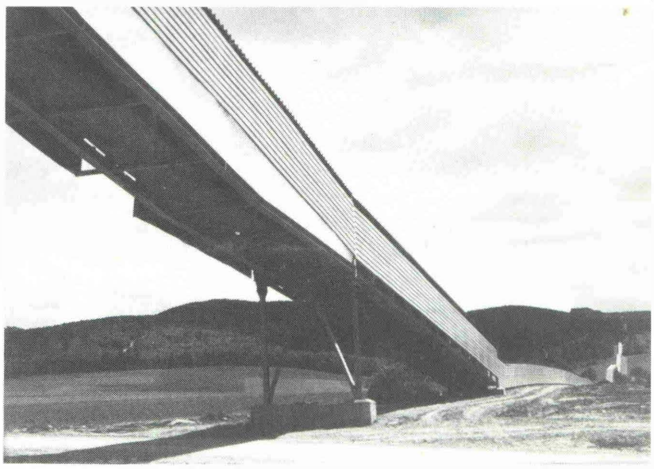

Fig. 2: Belt installation with changing inclination

working position, and these four increase to twelve when including start-up and stopping. As installations of this type are generally operated with a fixed take-up pulley, out of all the cases initially considered in isolation, the one with the maximum ensuing belt elongation should be looked at in detail. Twelve times six belt tension forces are to be allowed for and give a complete picture of possible belt stresses (Fig. 3).

If the equipment is to be operated as a bucket wheel scoop in the second working position with unchanged pretension, then another twelve operating states are possible and must

Continental - Application Engineering - Conveyor Belting

Conveyor Belt Calculation

Inquiry PROJECT DESIGN RECLAIMER

Feeding Data

Capacity in m3/h............	3000	Belt weight in kg/m..........	46.00
Bulk weight in t/m3.........	2.50	Weight of Idlers c.S. in kg/m	24.00
Belt speed in m/s...........	3.30	Weight of Idlers r.S. in kg/m	9.00
Conveying length in m.......	62.00	Friction Factor	
Section 1 Length in m.......	50.00	Carrying side	0.025
Section 2 Length in m.......	12.00	Return Side	0.025
Lift in m...................	12.00	Coefficient	2.000
Section 1 Lift in m.........	15.00		
Section 2 Lift in m.........	-3.00	Efficiency	0.95
Belt Width in mm............	1600	Idler Spacing in m...........	1.25

Results

Masses to be moved

	Section 1	Section 2
Carrying Side - empty Belt......	3500 kg	840 kg
Carrying Side - loaded Belt.....	35065 kg	8415 kg
Return Side.....................	2750 kg	660 kg

Kinetic Resistance

	Section 1	Section 2
Carrying Side - empty Belt......	7.6 kN	-1.1 kN
Carrying Side - loaded Belt.....	108.2 kN	-17.8 kN
Return Side.....................	-6.0 kN	1.5 kN
Secondary Resistance............	11.5 kN	

Driving Force and Power Input

Belt Loading	Driving Force	Power Input	
		Pulley	Motor
Empty Belt....................	1.9 kN	6.2 kW	6.6 kW
Section 1 loaded..............	114.0 kN	376.3 KW	396.1 kW
Loaded Belt...................	97.3 kN	321.1 KW	338.0 kW
Section 2 loaded..............	-14.8 kN	-48.9 KW	-46.4 kW

Worked out by DR.A Phone (0511)765 - 2544 Date 16.6.81

Fig. 3: Calculation of power and belt tensions with two sections

```
---------------------------------------------------------------------------------
  Inquiry PROJECT DESIGN                                               RECLAIMER
---------------------------------------------------------------------------------

            C a l c u l a t i o n   o f   B e l t   T e n s i o n s

---------------------------------------------------------------------------------
  Feeding Data
  ============
                          Power        Power         Braking     Arc of
                          installed    inserted      Force       Wrap
                          in kW        in kW         in kN       in degr.
                          ------------------------------------------------
  Head - Pulley  1 .....     400          400          -40          210
                 2 .....       0            0            0            0
  Tail - Pulley  3 .....       0            0            0            0
                 4 .....       0            0            0            0

  Startup Factor ......... 1.80     Tension/Tensioning in kN ...     0
  Coefficient Start/Brak.  0.35     Limit of Belt Sag in % ..... 1.00
                 Working....  0.35
  Manual Takeup
---------------------------------------------------------------------------------

  R e s u l t s   -   A l l   B e l t   T e n s i o n s   i n   k N
  ==================================================================
                                                      Startup  Working  Braking
                                                      -------------------------
                                   Slip - Head Pull. 1      5    -----       15
             Tail         Head                 Pull. 2      0    -----        0
  Carr.Side  T 6    Z 11   T 1            Tail Pull. 3      0    -----        0
             T 5           T 2                 Pull. 4      0    -----        0
  Ret. Side  T 4    Z 12   T 3     Belt Sag empty ....      0        7        0
                                   Belt Sag loaded ...      0      103        0

                  S t a r t u p     I   W o r k i n g    I   B r a k i n g
-----------------------------------I---------------------I---------------------
                Tail           Head I  Tail          Head I  Tail         Head
   Empty                            I                     I
   Belt          136     150    151 I  137    144     143 I  138    127    121
                 136            135 I  137            141 I  138           161
   ts =    1 s   136     138    135 I  137    143     141 I  138    159    161
   tb =    0 s                      I                     I
-----------------------------------I---------------------I---------------------
                Tail           Head I  Tail          Head I  Tail         Head
   1. Section                       I                     I
   loaded         97     239    238 I  103    223     222 I  141    124    119
                  97            100 I  103            108 I  141           159
   ts =    5 s    97     102    100 I  103    109     108 I  141    158    159
   tb =    0 s                      I                     I
-----------------------------------I---------------------I---------------------
                Tail           Head I  Tail          Head I  Tail         Head
   loaded                           I                     I
   Belt           97     242    230 I  104    224     206 I  133    151    108
                  97             99 I  104            109 I  133           148
   ts =    4 s    97     101     99 I  104    110     109 I  133    148    148
   tb =    1 s                      I                     I
-----------------------------------I---------------------I---------------------
                Tail           Head I  Tail          Head I  Tail         Head
   2. Section                       I                     I
   loaded        137     150    148 I  138    145     127 I  139    140    109
                 137            135 I  138            142 I  139           149
   ts =    1 s   137     138    135 I  138    144     142 I  139    149    149
   tb =    2 s                      I                     I
---------------------------------------------------------------------------------
```

Fig. 3: Calculation of power and belt tensions with two sections

also be taken account of in the calculations. The sequential calculation shows that in this situation there is a totally different distribution of the tension forces in the belt. This means the pretension may have to be readjusted.

3. Computer Application

For belt installations with long flights and numerous partial sections having differing inclinations the number of operating cases to be investigated and coordinated with one another increases substantially. The complex mathematical equations can be handled within a reasonable period of time only by making use of efficient EDP equipment.

The software needed for this purpose has to be custom-built in most cases and, because of the extensive data fields, requires ample storage capacity. Desk calculators, capable of being linked to form multi-location systems, are already meeting these requirements optimally (Fig. 4).

during start-up and stopping. The problems caused by irregularly changing belt stresses will be dealt with towards the end.

The lowest safety figures currently in force are obtained making due allowance for the theoretical time-tensile strength behaviour of any belt splice; they work out at $S = 2.8$ for steel cable belts and $S = 3.3$ for textile ply belts, even though the standard does not directly state these figures. Nevertheless, if these figures are taken as a basis then it is necessary to establish theoretically in advance all stresses which result from stationary and non-stationary operating states as well as from design features affecting belt guidance. In spite of the above this problem cannot, however, be solved readily.

There are several pointers and procedures for calculating the last mentioned stresses, generally defined as additional stresses and which, if applied correctly, ensure improved use of the belt tensile strength (Fig. 5).

Fig. 4: Using computers for conveyor belt calculations

Fig. 5: Convex conveyor belt curve

To minimize the amount of output data it is better to let the computer complete all necessary corrections without external interference until it comes up with the final solution. All critical limiting conditions, which led to the final result, should be shown though, thus facilitating any necessary specific input amendments aimed at further optimization.

4. Safety Margins

The safety margins stipulated in the draft to DIN 22101, which for maximum loading of the stationary belt already go down as low as $S = 6.7$, may in principle be used only together with belt tension forces determined in this way.

Nowadays safety factors of $S = 4.8$ are permitted for so-called non-permanent peak loads. The question remains unanswered, whether these low values may also be taken as a basis for operating states which, apart from when the belt starts up and stops, may also occur for limited periods when the belt is stationary for certain profiles and loads. This seems justified if stresses are taken into account collectively when designing the belt, i.e., extent and frequency of stressing must be included in the calculations. Belt tension distributions e.g. for increasing or decreasing loading can be equated in the probability of their occurrence with those

When calculating the additional stresses which occur, e.g., on troughing transitions of belts, on belts negotiating curves or where belt turnovers are involved, it is necessary to take account of the belt width as a further dimension affecting the outcome. The spatial run of the belt and the troughing geometry may cause a very uneven distribution of tension in the belt and consequently lead to increased strains.

Here no further reference is made to the influences resulting from belt loading and which, as has been proved, also give rise to non-uniform tensions and hence additional strains.

However, it must be pointed out that a complete investigation, as would be necessary for the application of minimum safety figures, would have to take these influences into account.

Even if the magnitude of these additional stresses is comparatively small, one has to be aware nonetheless that they have been omitted if, as an assumption in the following example, one proceeds from an initially even distribution of tension across the belt width (Fig. 6).

The example shows the investigation of belt strains at a trough transition whereby a practice-related procedure is employed. This, too, involves a certain amount of calculating so that the use of EDP is advisable.

C O N T I N E N T A L
RESEARCH AND DEVELOPMENT DEPARTMENT CONVEYOR BELTS

CALCULATION OF BELT STRAIN AT THE TRANSITION

PROJECT: EXAMPLE CONVEYOR: FLAT TO TROUGH

CALCULATION DATA:

BELT TYPE: ST 4000 SPLICE STRENGTH in N/mm: 3800

Dynamic modulus in N/mm....................	294000.00
Belt width in mm...........................	2400.00
Number of trough rollers...................	5
Roller lenght (center/internal) in mm......	520.00
Roller length (side/external) in mm........	520.00
Inner angle of troughing in degree.........	30.00
Outer angle of troughing in degree.........	55.00
Transition length in mm....................	4000.00
Pulley elevation in mm.....................	80.00
BELT TENSION at trough transition in kN....	425.00

RESULTS:

MEAN BELT TENSION in N/mm	177.08
MEAN SAFETY FACTOR	21.45

	BELT EDGE	BELT CENTER	MAXIMUM	MINIMUM
ELONGATIONS (cross section)	0.0031029	0.0000579	0.0031029	0.0000029
TENSIONS in N/mm	912.27	17.03	912.27	
SAFETY FACTORS	4.16	223.04		4.16

MINIMAL BELT TENSION (for compressionfree belt cross section) in kN 422.99

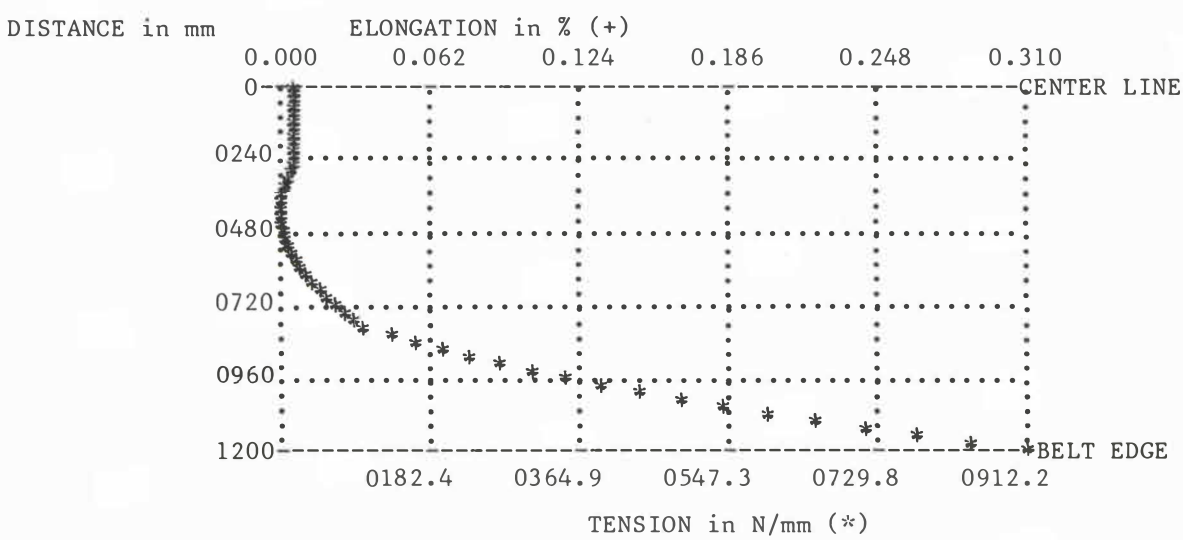

Fig. 6: Calculating the distribution of tension and elongation at trough transition a) without and b) with compression of belt

C O N T I N E N T A L
RESEARCH AND DEVELOPMENT DEPARTMENT CONVEYOR BELTS

CALCULATION OF BELT STRAIN AT THE TRANSITION

PROJECT: EXAMPLE CONVEYOR: FLAT TO TROUGH

CALCULATION DATA:

BELT TYPE: ST 4000 SPLICE STRENGTH in N/mm: 3800

Dynamic modulus in N/mm....................	294000.00
Belt width in mm...........................	2400.00
Number of trough rollers...................	5
Roller lenght (center/internal) in mm......	520.00
Roller length (side/external) in mm........	520.00
Inner angle of troughing in degree.........	30.00
Outer angle of troughing in degree.........	55.00
Transition length in mm....................	2800.00
Pulley elevation in mm.....................	140.00
BELT TENSION at trough transition in kN....	425.00

RESULTS:

MEAN BELT TENSION in N/mm	177.08
MEAN SAFETY FACTOR	21.45

	BELT EDGE	BELT CENTER	MAXIMUM	MINIMUM
ELONGATIONS (cross section)	0.0038835	0.0000363	0.0038835	-0.0002414
TENSIONS in N/mm	1141.77	10.69	1141.77	
SAFETY FACTORS	3.32	355.19		3.32

MINIMAL BELT TENSION (for compressionfree belt cross section) in kN 550.54

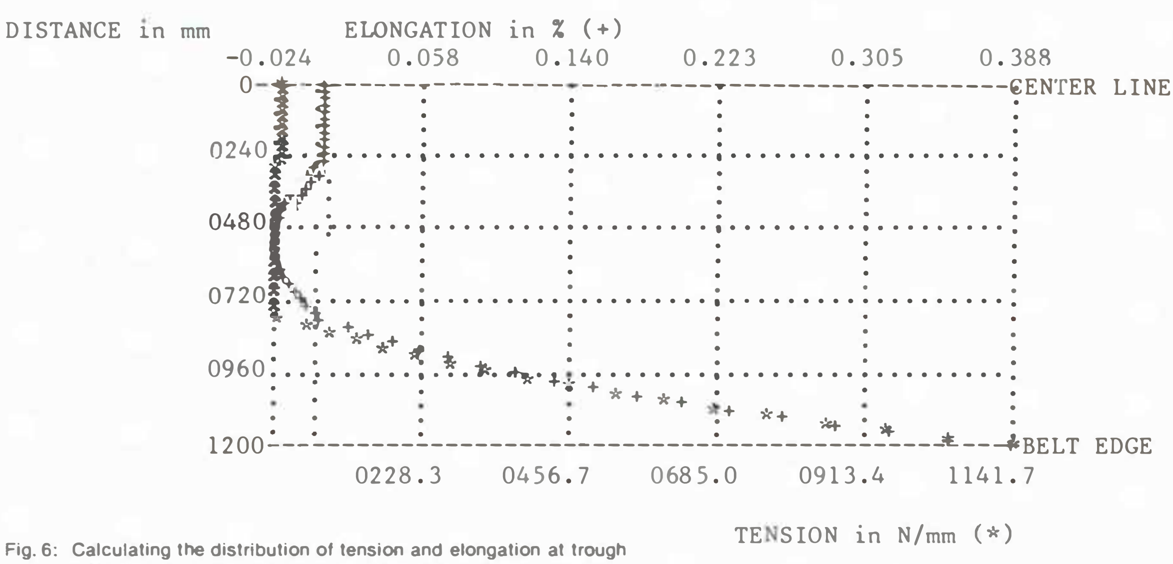

Fig. 6: Calculating the distribution of tension and elongation at trough transition a) without and b) with compression of belt

Apart from generally available belt geometry data, two input values require particular attention:

— The dynamic ply modulus, and
— the local belt pull force.

5. Dynamic Ply Modulus

Elongation properties play an important role in the belt calculation, so that the accuracy of the results depends on ascertaining the correct value of the dynamic ply modulus. This varies significantly for individual belt designs and can be found only approximately in laboratory examinations. When conducting these examinations the substantially higher strain frequency occurring here than in quasi stationary operations is to be taken into consideration.

The calculation method of additional elongations was developed from the screw line formula assuming a non-changing belt width. This approximates relatively closely to actual conditions. In this context various possibilities of two to five part trough shapes with or without crowning of the snub pulley can be opted for. The integration of additional tensions across the belt width and superposition with tensions resulting from the local belt pull force enables an analysis of the strains, thereby also taking into account that compressions but not any larger crushing stresses may occur under certain circumstances and in certain belt areas. Stresses of this nature should be avoided, if possible, although examinations of material have shown that even flexing amounting to more than 2 % belt compression and totalling over 2 million load cycles failed to destroy a Stahlcord® belt. The procedure outlined here enables a critical survey of operating conditions also in this respect (Fig. 7).

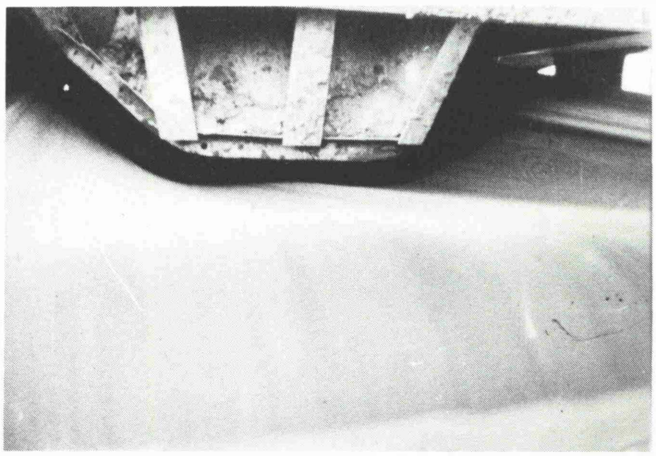

Fig. 7: Trough transition with incipient buckling of belt

It should be mentioned that supplementary studies regarding the shear deformation behaviour of conveyor belts have been carried out, the results of which may also be incorporated in the calculation. The belt code numbers obtained permit approximate consideration of the generally known fact that with certain limitations, the additional elongations propagate even outside the geometrical deformation area.

6. Local Belt Pull Forces

At this juncture one should to emphasize once again that prior to mathematically investigating the additional strains the local belt pull forces must be determined. According to the draft of DIN 22101 the permissible overall belt strain allowing for the time strength behaviour of the splice may also not be exceeded in any operating state or at any point of a belt conveyor.

Current studies, which are alluded to finally, deal with this area of minimum safety factors which is influenced by the time strength behaviour of the conveyor belt and in particular by its splice.

7. Time-Strength Behaviour

It is the aim of these as yet unfinished studies to devise a way of calculating these strains. Unfortunately this involves — even considering only the tensile strains in the conveyor belt — an aperiodically changing strain as a function of the design and mode of operation. Quasi stationary and intermittent operating states alternate in irregular sequence. This makes a mathematical treatment extremely difficult.

In similar cases the so-called operating strength can be determined in testing bays where actual strains can be simulated with some degree of accuracy. However, this is not so easy for conveyor belts of large-scale installations as this would require simulating up to several 10^5 load cycles with extremely high loads (Fig. 8).

Current tests — and even for those expensive installations are necessary — are therefore limited to ascertaining time strength behaviour under periodically alternating combined stressing. Such equipment can test within a reasonable period of time a steel cable belt of high tensile strength with several 100,000 alternating bending loads and several 10,000 load cycles under loads up to its nominal strength.

The result of such tests can be depicted for example in the form of a time strength graph which shows the number of load changes withstood for varying periodically changing load cycles

The influence exerted by stresses below the time strength graph has as yet not been determined. Among other things the current studies have to indicate a possible systematic mathematical treatment of these stresses.

Apart from an even better utilisation of the belt strength, the mathematical investigation of belt stresses permits important conclusions which serve the improvement of the conveyor belt as a high quality structural element. That is why an end to the development in this sector is not anticipated in the near future.

Fig. 8: Rotation testing bay designed to test belts under increasing load