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# Flow of Bulk Solids through Transfer Chutes of Variable Geometry and Profile

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**Der Förderstrom durch Übergabe-Schurren unterschiedlicher Geometrie und Querschnitte**
**Ecoulement de matériaux solides par des chutes de transfert à profil et géométrie variables**
**El flujo de sólidos a granel por rampas de transbordo de geometría y perfil variables**

可変形状・輪郭をもったトランスファーシュートを経由した粉体のフロー

不同形状和材型的溜槽在运输散装固料的流动

تدفق المواد الصلبة السائبة عبر مجاري النقل ذات التصميمات الهندسية والأشكال المختلفة


## Summary

The conditions for accelerated or 'fast' flow of bulk solids through transfer chutes of variable geometry and profile are discussed. A simplified chute design theory based on a lumped parameter dynamic model is presented. This model may be applied to chutes of arbitrary geometric profile but the emphasis is given to constant curvature and straight inclined chutes. Both rectangular and circular cross-sectional shapes are examined. The effects of variable Coulomb frictional drag resulting from variations in the contact or 'wetted' perimeter of the flowing stream in the acceleration zones are included in the model. Velocity dependent drag forces are also included. The solutions of the flow equations are described where the emphasis is on the determination of the stream thickness variation along the chute. The use of tapered chute sections is discussed.


## Nomenclature

$A$ Cross-sectional area of bulk solid in flowing stream (m²)
$B$ Width of rectangular chute. Diameter of circular chute (m)
$C$ Intergranular stress constant s²/m²
$C_1$ Inverse velocity Coulomb drag coefficient
$C_2$ Direct velocity Coulomb drag coefficient
$E_1$ Constant in equation (34)
$E_2$ Constant in equation (34)
$F_D$ Drag force (N)
$F_N$ Normal force (N)
$F_\mu$ Coulomb frictional drag force (N)
$F_v$ Velocity dependent drag force (N)
$g$ Acceleration due to gravity (m/s²)
$H$ Stream thickness (m)
$H_0$ Initial stream thickness (m)
$k$ Ratio of lateral to major normal pressure at the wall
$k_{EO}$ Effective linear pressure gradient down the wall surface at zero velocity
$k_{Ev}$ Effective linear pressure gradient down the wall surface during flow at velocity $v$
$M$ Mass (kg)
$n$ Normal co-ordinate
$Q_m$ Flowrate (t/h)
$R$ Radius of curvature (m)
$s$ Displacement along tangent (m)
$v$ Velocity (m/s)
$v_0$ Initial velocity (m/s)
$\phi_D$ Dynamic angle of repose
$\phi_R$ Angle of repose
$\xi$ Chute inclination angle
$\theta = (90° - \xi)$ Chute slope angle
$\varrho$ Density (kg/m³)
$\mu$ Coefficient of wall friction
$\mu_E$ Equivalent coefficient of friction
$\beta$ Velocity dependent drag coefficient
$\beta_v$ Viscous drag coefficient
$\lambda$ Surcharge angle
$\delta$ Angle defining 'wetted' or contact perimeter

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## 1. Introduction

Gravity-flow transfer chutes play an important role in bulk solids handling systems. While in many cases the chute may serve merely as a device for directing the material to the discharge point, it is important that attention be given to the design in order to obtain satisfactory flow characteristics.

The chute may also be used as a flow-control device and, in this regard, it is possible to design the chute to achieve certain desirable flow conditions. For example, it may be a requirement that the horizontal component of the exit velocity is as large as possible in order to obtain the maximum "throw" of the material. Alternatively, where the material discharges onto a conveyor belt, it is desirable that the horizontal component of the exit velocity be matched with that of the belt. In this way, the acceleration of the material entering the belt is minimized, yielding some saving in belt-driving power and a reduction in the belt wear. In other cases it may be desirable for the time of travel of the granular particles through the chute to be a minimum in order to achieve a favourable flow pattern.

The flow of non-cohesive bulk solids through discharge or transfer chutes has been studied in some detail, a selected list of references being included at the end of this paper. In general there are two classes of flow that occur, namely steady state flow and transient flow. The steady state flow corresponds to the condition of constant discharge rate and it is this condition that is of fundamental importance from the point of view of chute design. A knowledge of the transient flow characteristics is of particular importance in those cases where feed or discharge rates are varied during controlled flow, as in blending or mixing operations. This paper is concerned only with the steady flow case and for chutes of specified geometry, in particular straight inclined chutes and circularly curved chutes of constant radii.

It is based primarily on the papers by Roberts [8,9] and Roberts and Arnold [12] which presented analyses and design procedures for the flow of non-cohesive granular materials through chutes for a range of geometrical chute profiles but of rectangular cross-section. The concepts embodied in these papers have been further developed by Parlour [13] and Montagner [25]. The present paper extends the analysis to include chutes of circular cross-section as well as introducing into the analysis parameters which attempt to cover more explicitly the flow properties of bulk solids.

The problem of optimum flow through transfer chutes has been studied in some detail [14—17, 19—21, 23, 25]. While such problems may be seen to be more of 'academic' rather than practical use, the determination of optimum chute profiles to achieve specified performance criteria provides a useful basis for evaluating known chute profiles against the optimum. However, the determination of optimum chute profiles is excluded from this present paper.

Much of the research to date on the flow in chutes and channels has been confined to non-cohesive materials. It is clear that the flow behaviour of such materials is extremely complex and there is a need for more fundamental research in this area. Some progress has already been made and attention is drawn to the excellent paper by Savage [30] which has recently appeared. Savage has established the constitutive equations for the flow of cohesionless granular materials at high deformation rates and low stress levels. He has used this theory to study the fully developed, two dimensional, non-accelerating gravity flow in inclined chutes and vertical channels. Much progress has still to be made with respect to accelerated flows and the general class of cohesive bulk solids.

In the meantime there is a need for procedures for the design of transfer chutes of various geometrical form. It is believed that the theory presented in this paper provides an acceptable basis for chute design for both cohesionless and relatively free flowing cohesive bulk solids.

## 2. Flow Characteristics

### a) Curved Chutes — Modes of Flow

In the gravity flow of bulk solids through transfer chutes, two modes of flow are possible. These have been described in qualitative terms by Choda and Willis [6,7] and by Roberts [8,9] as 'fast' flow and 'slow' flow. Essentially the 'fast' and 'slow' flow terms are used to describe, respectively, the conditions of accelerated and uniform or non-accelerated flow that occur in practice.

The general flow conditions pertaining to transfer chutes are shown in Fig. 1. For the initial discussion it is assumed that the chutes are rectangular in cross-section. Referring to Fig. 1(a), the 'fast' or accelerated flow mode occurs when the material flows in contact with the chute bottom and side walls without contact with the top. The material on entering the chute first accelerates, increasing its velocity, but then, as a result of chute curvature and decreasing slope, it will pass into a deceleration zone causing the velocity to decrease. The resultant variation in stream velocity accounts for the variation in stream thickness along the chute. Where the material flows from a hopper into the chute, a vena contracta effect occurs and this results in an initial free-fall zone before contact is made with the chute bottom.

Ideally for stable fast-flow conditions, it is desirable that the chute be terminated at the "optimum" cutoff angle $\theta_{CO}$, as shown in Fig. 1(b); the optimum cutoff angle corresponds to the point of maximum velocity and minimum stream thickness. Where the chute continues beyond the optimum cutoff angle, the stream thickness will increase toward the end of the chute, as in Fig. 1(a).

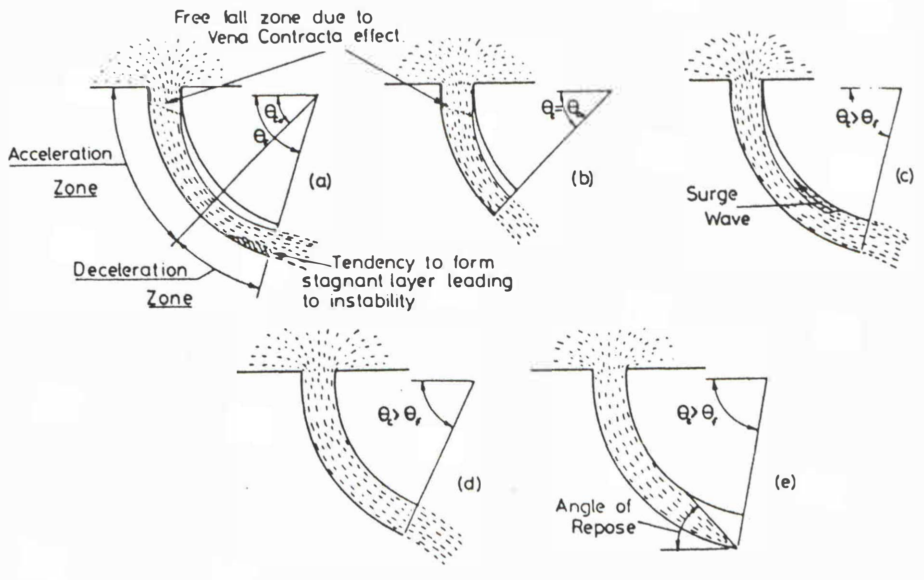


Fig. 1: Modes of chute flow:
(a) fast flow general case;
(b) fast flow, ideal case;
(c) transition from fast to 'slow' flow;
(d) slow flow;
(e) choked-flow condition, $\theta_c$ too large

Usually the build-up in thickness represents an unstable condition, since minor flow obstructions may cause a rapid deceleration of the stream with an increase in thickness near the end of the chute. The thickness may build up to such an extent that contact is made with the top surface of the chute. The stream is then slowed down considerably and a surge wave of material flows upstream as indicated in Fig. 1(c). The chute then becomes full and the material flows *en masse* at a uniform rate with contact being made with all four surfaces of the chute. This is the 'slow' flow mode and is shown in Fig. 1(d).

If the cutoff angle $\theta_C$ is too large, a flow obstruction, even though only momentary, will cause a cessation of fast flow with consequent filling of the chute without further discharge. This is the choked-flow condition shown in Fig. 1(e). It is important, therefore, that for the maintenance of fast flow, the cutoff angle should not be greater than the limiting angle $\theta_f$ which depends on the frictional properties of the grain on the chute surfaces.

Under fast flow conditions, the discharge rate of the grain is governed only by the aperture in the bin or hopper and is not influenced by the chute. On the other hand, when slow flow prevails, the chute acts as an extension of the hopper and the chute geometry has a marked effect in reducing the flow rate. This is illustrated in Fig. 2 which shows the form of a typical discharge curve for a bin fitted with a circularly curved chute of increasing cutoff angle. For the reasons stated, 'fast' flow is the ideal flow pattern.

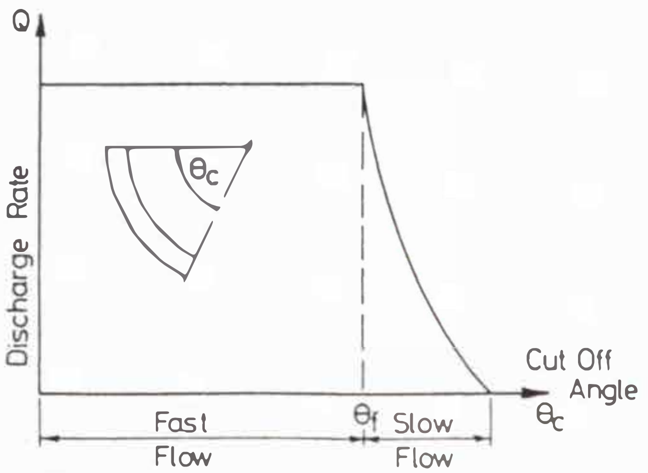


Fig. 2: Typical discharge graph for circularly curved chute fitted to flat bottom bin

**b) Straight Inclined Chutes**

'Fast' flow is automatically achieved in the case of straight inclined chutes of constant cross sectional area, provided the chute slope is sufficient to permit accelerated flow in the presence of various drag forces. In such cases the material will flow with increasing velocity until some terminal velocity is reached. The corresponding stream thickness variation is one which shows a gradual thinning down until the steady state constant thickness is reached corresponding to the terminal velocity. If a terminal velocity is not reached during the period of transit through a straight inclined chute, the stream thickness variation will continue to show a decrease along the path. This is illustrated in Fig. 3.

In view of its simplicity in shape as well as its importance to practical applications, the straight chute provides a useful basis for a more detailed and fundamental study of the characteristics of chute flow. Parlour [13] examined in

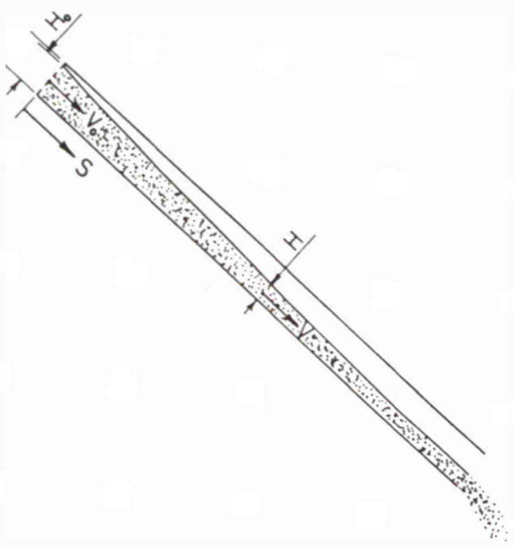


Fig. 3: Flow of bulk solid through straight inclined chute

some depth the flow of non-cohesive granular materials through straight chutes of rectangular cross section. One phase of this work was devoted to the study of the transition from 'fast' to 'slow' flow. It was shown that 'slow' flow did not necessarily depend on the material building up in thickness so as to make contact with the top surface of the chute, as previously described for the curved chute. In fact 'slow' flow was shown to occur in open channels at low inclination angles, close to the angle of repose of the material on the chute surface. Under these conditions the discharge rate is controlled by the chute rather than the bin orifice.

Savage [30] also studied chutes inclined at relatively low angles and showed that despite the initial depth of material at entry to the chute, the stream approached a uniform thickness. Savage explained that for steady, fully developed, constant depth flow (previously referred to as slow flow) the chute inclination angle $\xi$ measured from the horizontal should be within the range

$$\phi_R < \xi < \phi_D$$

where $\phi_R$ = angle of repose

and $\phi_D$ = dynamic internal friction angle

The explanation given by Savage is supported by the observations of Parlour [13]. Parlour noted that for millet seed and polythene particles there existed lower and upper chute inclination bounds differing by about 4° within which fully developed flow occurred. Outside this narrow range fully developed flow is not possible.

Savage reported on the phenomenon of surge waves and granular jumps which are analogous to hydraulic jumps and which could be formed when obstructions are placed at the downstream end of the chute. The possibility for the spontaneous formation of granular jumps also exists. It is shown that the occurrence of granular jumps is related to the upstream Froude Number. The phenomenon of granular jumps provides an explanation of the transition from the 'fast' to 'slow' mode referred to previously in the case of the curved chute (see Fig. 1(c)).

#### c) Chute Flow Theory

As an aid to understanding the mechanism of 'fast' flow, Roberts [9] used high speed movie photography to examine the flow patterns at particular sections of the flowing stream. The study was performed in a curved chute of rectangular cross-section using a free-flowing non-cohesive granular material. It was established that the grains moved in substantially parallel paths with the path velocity increasing across the stream, as shown in Fig. 4.

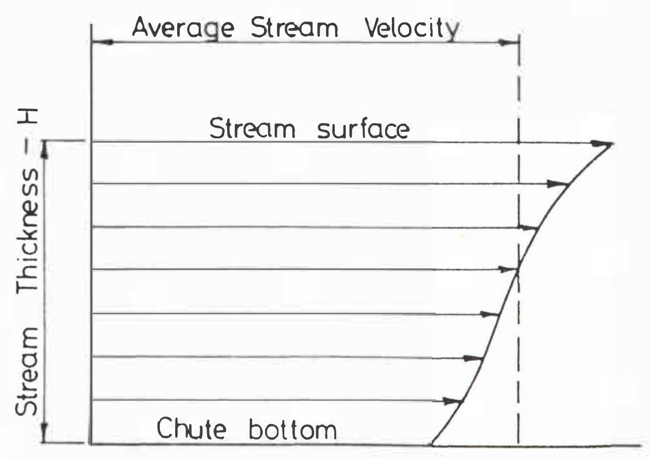


Fig. 4: Typical grain system velocity distribution for fast flow

The flow is predominantly a sliding action of grain on chute surface and inter-granular sliding, although a certain amount of rolling action was observed to take place. The grain path velocity increases across the stream with the grains on the surface travelling the fastest. However, for most instances the variation in velocity across the thickness or depth of the flowing stream is small compared with the average stream velocity.

Although some relative motion does occur between grain layers, the friction losses due to this motion were found to be only approximately 9 % of the total frictional losses. For the rectangular cross-section chute, the major portion of the friction losses are due to grain sliding against the chute bottom, these losses being in the order of 82 %; the remaining losses ($\simeq$9 %) arise due to grain sliding against the side walls of the chute.

In view of these observations it was found that the steady-state flow could be satisfactorily studied by regarding the material as a lumped particle moving with the average velocity of the stream layers. Using the average velocity compensates in some way for the inter-granular friction losses.

The velocity variations in two directions, across the depth and across the width of the flowing stream, have been determined by Savage [30] by both analytical and experimental methods for the fully developed, uniform flow case.

#### d) Cohesive Materials

While much work on chute flow has been performed using free flowing granular type materials, the flow of fine powders and cohesive bulk solids also needs to be examined.

Some preliminary studies were performed on alumina powder flowing from a hopper through a straight inclined chute of circular cross-section. The test rig is shown schematically in Fig. 5. At the point of entry to the straight chute from the curved bend the stream velocity was low and the stream thickness correspondingly high. Although it was difficult to observe precisely, the flow at the upstream end appeared to commence as a series of block-wise shears with each block elongating and reducing in thickness as the velocity increased. This produced a step-wise stream surface, in the

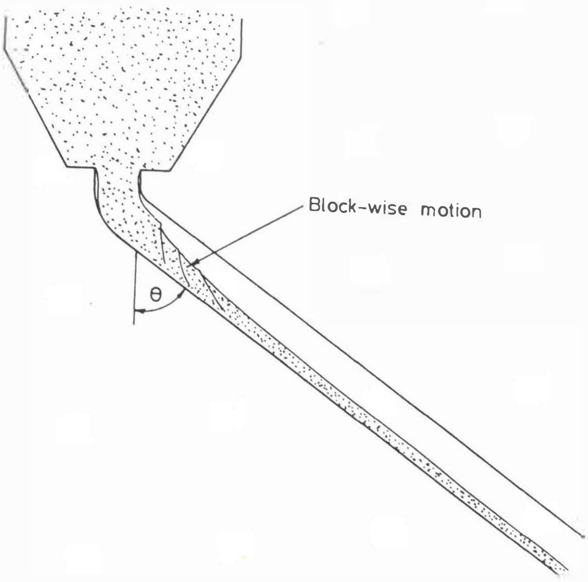


Fig. 5: Test rig for chute flow studies — flow characteristics for alumina powder.
Test chute: 0.073 m dia. x 2.0 m long

upstream region. A short distance down the chute where the stream velocity had increased sufficiently and the thickness had reduced, the material flowed as a uniform stream with a gradually reducing thickness. In view of the relative motion at the interface of the flowing stream and adjacent air in the unfilled space of the chute, some particles were observed to be in suspension with air. This effect has also been observed by Ridgway and Rupp [11] and by Savage [30].

The upstream flow behaviour observed for the alumina is characteristically different to that of non-cohesive granular type materials. In the latter case the material flows with relative motion between the grains being predominantly in the longitudinal direction leading to a gradual thinning down of the stream along the total length.

It should be noted that the block-wise motion observed for the alumina is similar to that observed by Rademacher [29] when he studied the discharge of cohesive bulk materials from the rotating buckets of a bucket elevator.

## 3. A Dynamic Model for Chute Flow

#### a) Lumped Parameter Model

The design of transfer chutes for the accelerated or 'fast' flow condition is dependent on a knowledge of the stream thickness variation along the chute. In order to avoid blockage and possible transition to the fully developed or 'slow' mode of flow, it is desirable to terminate the chute well before the point at which rapid increase in thickness of the flowing stream will occur.

Restricting the analysis initially to the two-dimensional case and following the theory established by Roberts [9], the lumped parameter model illustrated in Fig. 6 may be used to describe the motion. In view of the characteristics of 'fast'

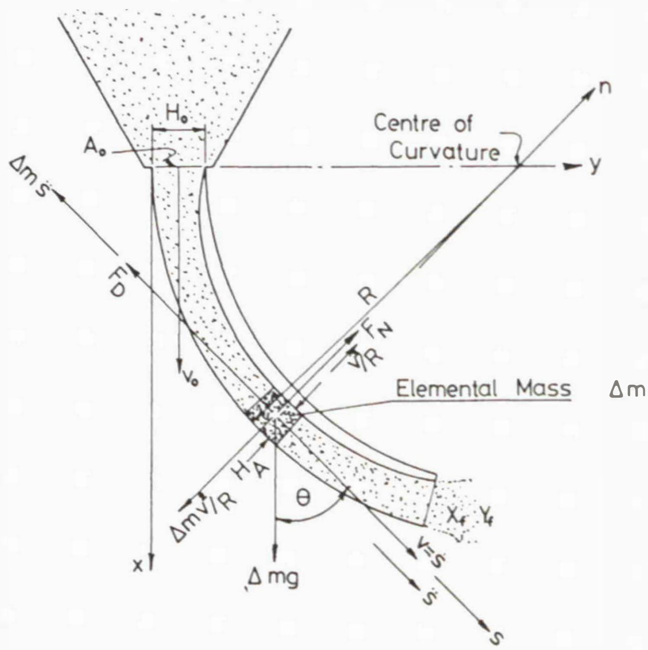


Fig. 6: Chute flow model

flow previously discussed, certain assumptions can be made which simplify the analysis. The principal assumption is that the material behaves as a continuum with the mass flow rate being constant throughout the flow. This assumption has been shown to be valid in cases where the material is free-flowing and the grain stream thickness is small in relation to the radius of curvature of the chute. Under these conditions, the interactionary effects between particles are considered to be negligible; the major factors governing flow are the chute curvature and frictional drag along the internal chute walls.

Referring to Fig. 6, the various forces acting on an element of the bulk material are as shown. While the particular co-ordinate system chosen will depend on the selected chute geometrical profile [9], in this case a moving co-ordinate system is chosen with tangential "$s$" and normal "$n$" components. This co-ordinate system is particularly relevant in view of the constant curvature and straight chutes which are commonly used in practice.

Since for design purposes the objective is to establish the stream thickness $H = f(s)$ or the stream cross-sectional area $A = f(s)$, then it is first necessary to determine the velocity variation along the chute. From the model it can easily be seen that by resolving the forces in the tangential direction the following equation of motion applies

$$\ddot{s} = g \cos\theta - \frac{F_D}{\Delta m} \qquad (1)$$

where

$\ddot{s} = \dot{v}$ = acceleration along the tangent
$g$ = acceleration due to gravity
$\theta = \frac{dy}{dx}$ = chute slope
$\Delta m$ = mass of element
$F_D$ = drag force

Since $\ddot{s} = v\frac{dv}{ds}$ then equation (1) becomes

$$\frac{dv}{ds} = \frac{g}{v}\cos\theta - \frac{F_D}{v\Delta m} \qquad (2)$$

Alternatively the acceleration along the tangent may be expressed as

$$\ddot{s} = R\ddot{\theta} + \dot{R}\dot{\theta}$$

where

$R$ = radius of curvature of the chute at the position indicated by $\theta$

Also since $\ddot{\theta} = \dot{\theta}\frac{d\dot{\theta}}{d\theta}$

then an alternative form of equation (2) is

$$\frac{d\dot{\theta}}{d\theta} = \frac{g}{R\dot{\theta}}\cos\theta - \frac{F_D}{R\dot{\theta}\,\Delta m} - \frac{\dot{R}}{R} \qquad (3)$$

Before any solution of (2) or (3) may be obtained, it is first necessary to establish an expression for the drag force $F_D$. Once the expression for $F_D$ is known, the solution of (2) or (3) can be readily obtained. That is

$$\left.\begin{aligned} v &= f(s) \quad \text{or} \\ v &= R\dot{\theta} = f(\theta) \end{aligned}\right\} \qquad (4)$$

**b) Continuity of Flow**

While the bulk density $\varrho$ at any section of the flowing stream would show some variation with depth, this variation is small and it is convenient to assume a constant average value. Such an assumption is consistent with that reported by Augenstein and Hogg [27] and by Savage [30].

Assuming uniform mass flow, then the equation for continuity of flow is

$$Q_m = \varrho A v = \text{constant} \qquad (5)$$

Although some variation in the density would no doubt occur along the chute, it is likely that in the case of thin stream motion this variation is small. Hence, for simplicity $\varrho$ is assumed to be constant along the path. Therefore from (5) it follows that

$$A\,v = \text{constant}$$

or

$$A\,v = A_o v_o \qquad (6)$$

where

$A_o$ = initial cross-sectional area of the flowing stream at the point of entry of the chute
$v_o$ = corresponding initial velocity

Since, for a given chute cross-section, the stream thickness $H$ is a function of the area $A$, that is

$$H = f(A) \tag{7}$$

then from (2) or (3) together with (7) the stream thickness variation can be determined.

The initial cross-sectional area can be computed from the mass flow rate. That is

$$A_o = \frac{Q_m}{\varrho V} \tag{8}$$

**c) Drag Force**

*(i) Components of Drag Force*

The drag force $F_D$ may be considered to be composed of two components, that due to Coulomb friction and that due to velocity dependent resistance resulting from air drag on the surface of the flowing stream and air permeation through the mass. That is

$$F_D = F_\mu + F_v \tag{9}$$

Here $F_\mu$ is the Coulomb frictional drag and is given by

$$F_\mu = \mu_E F_N \tag{10}$$

where $\mu_E$ = equivalent friction coefficient

$\mu_E$ takes into account frictional drag on the internal chute walls. It is a function of the wall pressure distribution and the 'wetted' or contact perimeter.

$F_v$ is the velocity dependent drag and can be expressed in generalised form as

$$F_v = \beta\, \phi\, (v) \tag{11}$$

where $\beta$ = drag coefficient.

*(ii) Equivalent Friction Coefficient $\mu_E$ — Chutes of Rectangular Cross Section*

In the work of Roberts [9] in which the flow through chutes of rectangular cross-section was examined, the pressure distribution as shown in Fig. 7 was assumed. As indicated, the pressure against the side walls was taken to be directly related to the hydrostatic pressure; that is, a linear increase with depth. The pressure across the chute bottom was assumed to be uniform.

Based on this pressure distribution it was shown that the equivalent Coulomb friction coefficient $\mu_E$ is given by

$$\mu_E = \mu \left(1 + k\, \frac{H}{B}\right) \tag{12}$$

where

$\mu$ = coefficient of wall friction
$k$ = ratio of lateral to major normal pressure at the wall, based on the assumed linear pressure gradient
$B$ = chute width

Experimental verification of equation (12) was obtained by Roberts by measuring the angle of repose (and hence the equivalent friction coefficient $\mu_E$) of slugs of millet seed of different $H/b$ ratios flowing in a straight rectangular perspex channel. In each case the material was retained in a cage consisting of two end plates tied together and which were a loose fit in the channel.

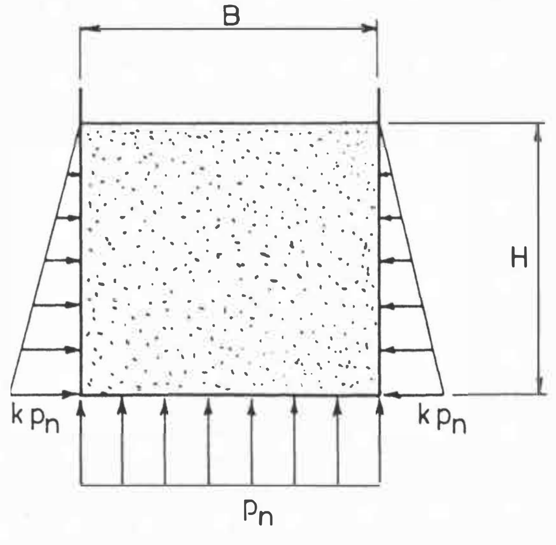


Fig. 7: Assumed pressure distribution around boundaries of chute of rectangular cross-section (Roberts theory)

More recently Savage [30] used the same procedure as Roberts to determine $\mu$ and $k$ in order to compare his own findings with those predicted by Roberts based on the lumped with $\mu_E$ given by equation (12). Savage showed that for low chute angles close to those pertaining to fully developed flow, the bed depths or stream thicknesses predicted by the theory of Roberts were considerably less than the measured values. However, at higher bed slopes the experimental flows were found to accelerate over the length of the chute, giving closer agreement with the predictions of Roberts.

It needs to be noted that the angle of repose method used to determine $\mu$ and $k$, as previously described, while giving correct values of $\mu$, gives values of $k$ lower than those that would occur in a flowing stream. In the latter case the relative motion between individual bulk solid particles would accentuate the lateral pressure normal to the walls rather more than in the angle of repose test where the slugs of material just commence sliding without inter-particle relative motion. Furthermore the inclusion of the velocity dependent drag force of equation (11) in the flow model provides a more realistic representation of the flow. This is an additional resistance which further limits the reduction in stream thickness.

In an experimental study using straight inclined chutes of rectangular cross-section, Parlour [13] determined the pressure distribution around the chute boundary during flow. A typical pressure distribution is shown in Fig. 8. The results, which apply to millet seed, show that the pressure along the chute walls increases approximately linearly with depth over the upper portion of the grain stream. At greater depths below the surface the pressure increases non-linearly at a more rapid rate, reaching a peak value at the chute corner.

The results shown in Fig. 8 indicate that the value of $k_v$ relating lateral pressure at the wall to the average normal pressure during flow with velocity $v$ varies with the stream depth. However, it is possible to assign an equivalent linear pressure gradient coefficient $k_{Ev}$; for the particular distribution shown in Fig. 8 the value of $k_{Ev}$ is 0.78, which is more than double the 'static' value from the angle of repose test.

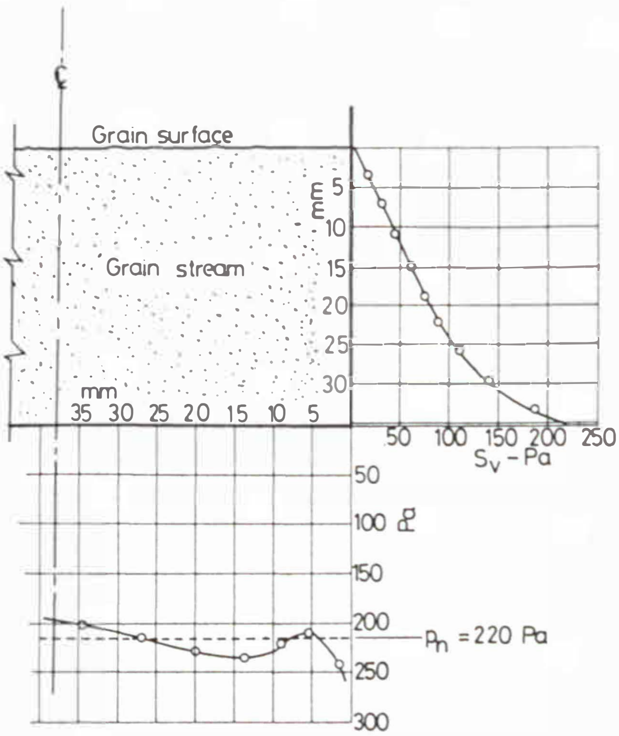


Fig. 8: Pressure distribution in rectangular chute using Hungarian millet

Parlour showed that the coefficient $k_{Ev}$ varies with both the total stream thickness and stream velocity. From studies performed with millet seed, polythene particles and wheat, he proposed the following expression for $k_{Ev}$

$$k_{Ev} = k_{EO}(1 + C v^2) \quad (13)$$

where

$k_{EO}$ = effective linear pressure gradient down the wall surface at zero velocity

$C$ = intergranular stress constant, sec²/m²

Average values of C determined by Parlour are

$C = 0.2314$ sec²/m² for millet
$C = 0.2368$ sec²/m² for polythene
$C = 0.0344$ sec²/m² for wheat

Subsequent analysis of the data suggests that these values of $C$ may be slightly underestimated.

With $k_{Ev}$ given by equation (13), equation (12) for the equivalent friction coefficient may now be refined

$$\mu_E = \mu\left[1 + k_{EO}\frac{H}{B}(1 + C v^2)\right] \quad (14)$$

From equation (6), for a rectangular cross-section chute, it follows that

$$v H = v_0 H_0 \quad (15)$$

where

$H_0$ = initial stream thickness (m)
$v_0$ = initial velocity (m/sec)

Hence (14) can be expressed as

$$\mu_E = \mu\left[1 + \frac{C_1}{v} + C_2 v\right] \quad (16)$$

where

$$C_1 = k_{EO}\frac{v_0 H_0}{B}$$

and

$$C_2 = k_{EO}\frac{v_0 H_0 C}{B}$$

Equation (16) shows that for the rectangular chute the equivalent friction coefficient involves three components, a constant value $\mu$, a component that varies inversely with the stream velocity and a component that varies directly with the stream velocity.

*(iii) Equivalent Friction — Chutes of Circular Cross-Section*

As in the case of rectangular chute cross-sections, the determination of the equivalent friction coefficient $\mu_E$ for circular cross-sections is dependent on the knowledge of the normal pressure distribution around the contact or 'wetted' perimeter of the chute wall. Referring to Fig. 9, it can readily be shown that the general relationship for $\mu_E$ is

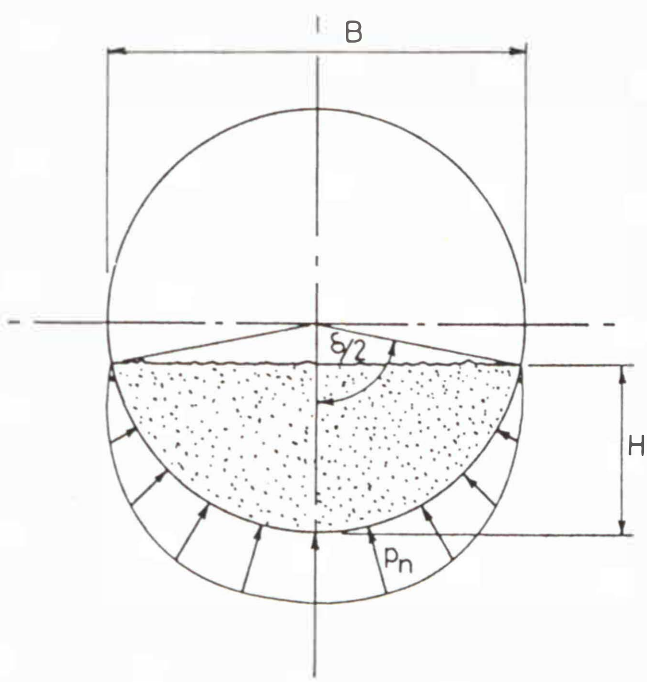


Fig. 9: Normal pressure distribution around circular chute

$$\mu_E = \frac{\mu \int_0^{\delta/2} p_n \, d\theta}{\int_0^{\delta/2} p_n \cos\theta \, d\theta} \quad (17)$$

where $\delta$ = angle defining the 'wetted' perimeter and $p_n$ = normal pressure.
Equation (17) will have a lower bound value $\mu_E = \mu$ in the limit as $\delta \rightarrow 0$.

Since, for a circular chute, the ratio of stream thickness $H$ to chute diameter $B$ is a function of the angle $\delta$ equation (17) may be readily expressed in the functional form

$$\mu_E = \mu \left[ 1 + k_v f \left( \frac{H}{B} \right) \right] \tag{18}$$

where $k_v$ is a pressure distribution factor.

Experimental studies were carried out in which the angle of repose of slugs of material flowing through a chute of circular cross-section was examined in relation to variations in $H/B$. The results show that $\mu_E$ is relatively insensitive to changes in $H/B$, these observations being also confirmed by the solution equations of (17) and (18) for assumed expected normal pressure distributions. This behaviour is in contrast to rectangular chute cross-sections where variations in $H/B$ have a significant effect on $\mu_E$; in this case $B$ is the chute width. On the basis of the observations for the circular cross-section, it has been found that the assumption of a linear variation of $\mu_E$ with $H/B$ is quite acceptable for the case of accelerated flow in which the stream thickness is reducing. It is also acceptable for cases where the stream thickness is increasing provided $H < B/2$, as will be explained later.

The assumption of a linear function for $\mu_E$ simplifies the analysis and permits the application of equation (14) to chutes of circular cross-section. In this case $B$ denotes the chute diameter and the coefficients $k_{EO}$ and $C$ are those relating to circular cross-sections; they will normally be lower than the values obtained for rectangular cross-sections. From this discussion it may be suggested that equation (14) will hold for accelerated flow through chutes of arbitrary cross-sections where $B$ is the equivalent 'hydraulic' diameter.

*(iv) Velocity Dependent Drag*

The drag force component $F_v$ expressed in general form in equation (11) may, in many cases, be approxiamated by a viscous drag force.

That is

$$F_v = \beta_v \Delta m\, v \tag{19}$$

where $\beta_v$ is the viscous drag coefficient.

**d) Generalised Equation of Motion**

From the foregoing, the generalised drag force may be expressed as

$$F_D = \mu \left[ 1 + k_{EO} \frac{H}{B} (1 + C v^2) \right] F_N + \beta_v \Delta m\, v \tag{20}$$

Referring to Fig. 6, the normal force is given by

$$F_N = \Delta m \left( g \sin\theta + \frac{v^2}{R} \right) \tag{21}$$

Substitution of (20) and (21) into (2) and (3), respectively, yields

$$\frac{dv}{ds} = \frac{g}{v} \cos\theta - \frac{\mu}{v} \left[ 1 + k_{EO} \frac{H}{B} (1 + C v^2) \right] \left[ g \sin\theta + \frac{v^2}{R} \right] - \beta_v \tag{22}$$

and

$$\frac{d\dot{\theta}}{d\theta} = \frac{g}{R\dot{\theta}} \cos\theta - \frac{\mu}{R\dot{\theta}} \left[ 1 + k_{EO} \frac{H}{B} (1 + C R^2 \dot{\theta}^2) \right] \left[ g \sin\theta + R \dot{\theta}^2 \right] - \beta_v + \frac{\dot{R}}{R} \tag{23}$$

## 4. Chutes of Specified Geometry

**a) Equation of Motion**

While equations (24) and (25) may be applied to plane chutes of arbitrary geometrical shape, in practice two chute profiles are of importance, namely, the straight inclined chute and circularly curved chute.

*(i) Straight Inclined Chute*

In this case $R = \infty$ and equation (22) becomes

$$\frac{dv}{ds} = \frac{g}{v} \left\{ \cos\theta - \mu \sin\theta \left[ 1 + k_{EO} \frac{H}{B} (1 + C v^2) \right] \right\} - \beta_v \tag{24}$$

*(ii) Circularly Curved Chute*

For circularly curved chutes of constant radius $R$, it follows that $\dot{R} = 0$. Hence equation (23) becomes

$$\frac{d\dot{\theta}}{d\theta} = \frac{1}{R\dot{\theta}} \left\{ g \cos\theta - \mu \left[ 1 + k_{EO} \frac{H}{B} (1 + C R^2 \dot{\theta}^2) \right] [g \sin\theta + R \dot{\theta}^2] \right\} - \beta_v \tag{25}$$

Equations (24) and (25) may be readily solved numerically. However, before any solutions can be obtained, it is first necessary to establish expressions relating the velocity $v$ as a function of the stream thickness $H$.

**b) Stream Thickness Variation**

In view of their practical importance, the rectangular and circular chute cross-sections are examined.

*(i) Chutes of Rectangular Cross-Section*

Referring to Fig. 10 and assuming a parabolic surcharge with surcharge angle $\lambda$, $\lambda$ may be positive or negative depending on whether the surcharge is positive (convex) or negative (concave). An expression for the cross-sectional area may be obtained.

$$A = A_1 + A_2 = BH_1 + \frac{B^2 \tan\lambda}{\lambda} \tag{26}$$

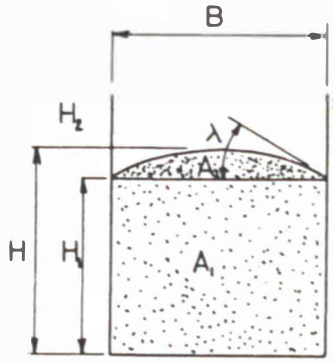


Fig. 10: Cross-section of flowing stream in rectangular chute

and

$$H = H_1 + B \tan \lambda \tag{27}$$

*(ii) Chutes of Circular Cross-Section*

Referring to Fig. 11 and again assuming a parabolic sur-

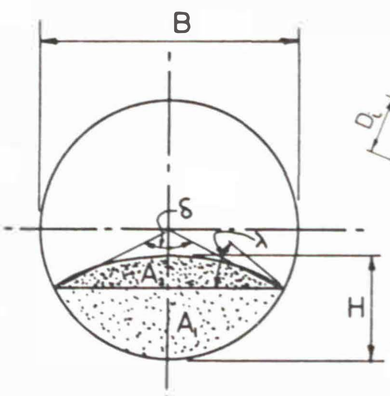


Fig. 11: Cross-section of flowing stream in circular chute

charge with surcharge angle λ and using angle δ to define the 'wetted' or contact perimeter, it may be shown that:

$$A = A_1 + A_2$$

$$A = \frac{B^2}{4}\left[\frac{\tan\lambda}{3}(1-\cos\delta) + \left(\frac{\delta-\sin\delta}{2}\right)\right] \tag{28}$$

From the geometry it may also be shown that:

$$H = B \sin\frac{\delta}{2}\tan\lambda + \frac{B}{2}\left(1-\cos\frac{\delta}{2}\right) \tag{29}$$

*(iii) Stream Thickness*

The combination of the computed velocity distribution $v = f(s)$, together with the appropriate equations for $A$ and $H$, enables the ratio $\frac{H}{H_0} = f(s)$ to be determined. Here $H_0$ is the initial stream thickness at entry, as previously defined.

For stable 'fast' flow, it is most desirable that $\frac{H}{H_0} < 1$ and that $\frac{H}{H_0}$ decreases as $s$ increases.

**c) Chutes of Circular Cross-Section — General Remarks**

From a practical point of view, chutes of circular cross-section offer many advantages. However, for the maintenance of stable fast-flow conditions, they require special consideration, as illustrated in Fig. 12. In the acceleration zone during

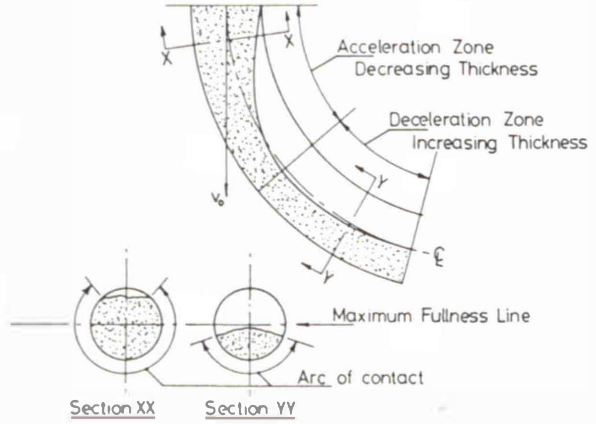


Fig. 12: Stable fast flow through chute of circular cross-section

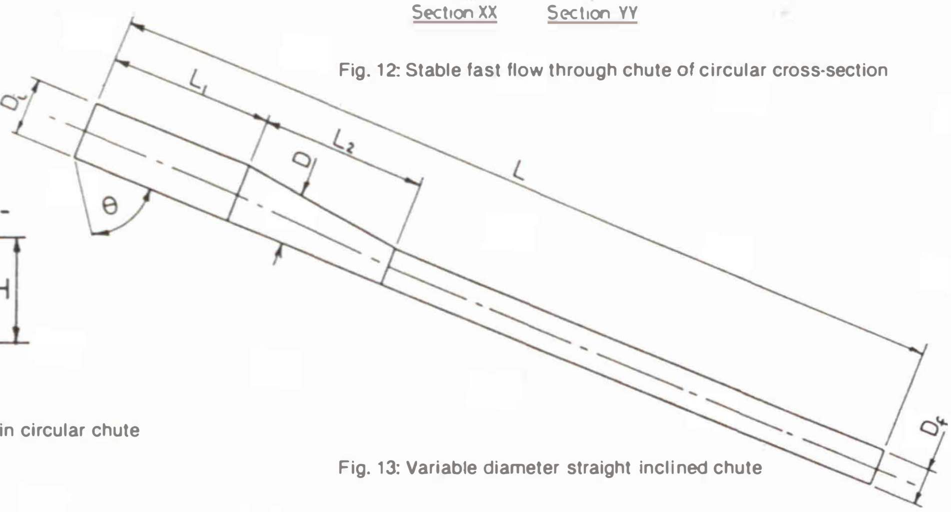


Fig. 13: Variable diameter straight inclined chute

which the stream thickness is decreasing, it is possible for the chute to be more than half filled at any point, with the arc of contact exceeding half the chute circumference. However, in the deceleration zone where the grain stream thickness is increasing, it is suggested that the arc of contact should not be allowed to exceed half the chute circumference. Effectively, this means that the chute fullness at any point in the deceleration zone should always be less than half.

In the case of straight inclined chutes, it is possible to reduce the chute diameter by using a converging or tapered section, as illustrated in Fig. 13. It is important that the convergence angle be kept quite small (< 2°) in order that the *H/D* ratio be kept within desirable limits. In addition it is desirable to avoid 'granular' jumps and flow instabilities. In the case of long chutes, reducing the diameter can yield some cost savings. It should be noted that it is very desirable to maintain the chute bottom collinear in the tapered section, as illustrated in Fig. 13. The design of chutes of the form shown in Fig. 13 may be performed by utilizing equations (24), (28) and (29).

### d) Approximate Closed-Form Solutions of Flow Equations

If the velocity dependent drag component $F_v$ is neglected, then from equations (10) and (21) the drag force can be expressed as

$$F_D = \mu_E \Delta m \left( g \sin\theta + \frac{v^2}{R} \right) \tag{30}$$

It should be noted that assumptions of $\mu_E$ = constant and $F_v = 0$ are satisfactory in cases of short length chutes. The flow is assumed to be fully accelerated.

The generalised flow equation from (2) can be expressed as

$$\frac{dv}{ds} = \frac{g}{v} \cos\theta - \frac{\mu_E}{v} \left( g \sin\theta + \frac{v^2}{R} \right) \tag{31}$$

Again, the straight inclined chutes and the circular curved constant radii chutes are of particular interest.

*(i) Straight Inclined Chutes*

In this case, with $R = \infty$, equation (31) becomes

$$\frac{dv}{ds} = \frac{g}{v} (\cos\theta - \mu_E \sin\theta) \tag{32}$$

Here $\theta$ is constant.

A closed form solution of the form $s = f(v)$ is readily available for the case when $\mu_E$ is given by

$$\mu_E = \mu \left( 1 + \frac{C_1}{v} \right) \tag{33}$$

This equation is readily obtained from equation (12).

Substituting in (31) and solving yields

$$s = \frac{v - v_0^2}{2E_1} + \frac{E_2}{(E_1)^2} + (v - v_0) + + \frac{E_2}{(E_1)^3} \ln \left[ \frac{E_1 v - E_2}{E_1 v_0 - E_2} \right] \tag{34}$$

where

$E_1 = g\,(\cos\theta - \mu \sin\theta)$
$E_2 = \mu g\, C_1 \sin\theta$

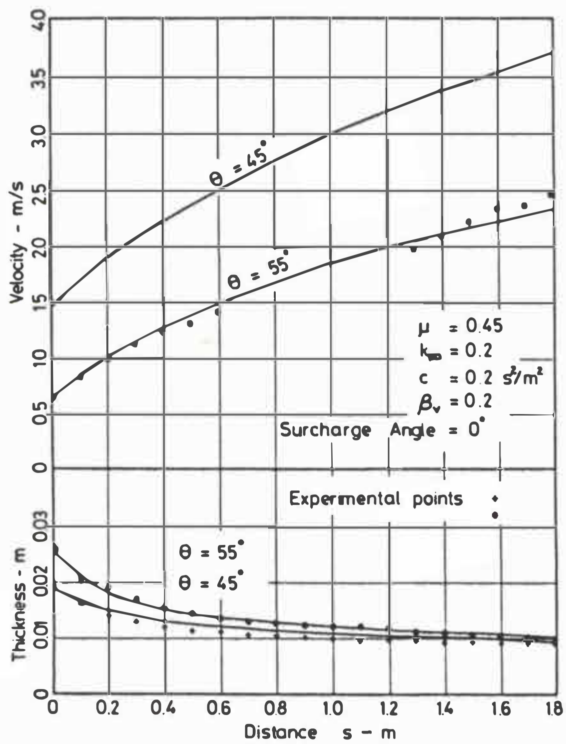


Fig. 14: Stream thickness and velocity variation for powdered alumina

As can be observed, the factor $E_1$, which is constant for a particular chute inclination, represents the acceleration of a single particle down the chute.

In the work of Roberts and Arnold [9,10], it was established that satisfactory solutions to flow equations can be achieved by assuming $\mu_E$ is constant at a value corresponding to the average stream thickness along the chute. With this assumption, the acceleration down the straight inclined chute is constant and the velocity distribution is given by

$$v = \sqrt{v_0^2 + 2gs(\cos\theta - \mu_E \sin\theta)} \tag{35}$$

*ii) Circularly Curved Chutes*

Assuming $\mu_E$ constant and re-writing (31)

$$\frac{d\dot\theta}{d\theta} = -\mu_E\dot\theta - \frac{g}{R\dot\theta} (\mu_E \sin\theta - \cos\theta) \tag{36}$$

As shown by Roberts [9], this equation has the same form as Bernoulli's equation, for which a known solution exists.

The solution of (36), noting that $v = R\dot\theta$, is

$$v = \sqrt{\frac{2gR}{4\mu_E^2 + 1} [\sin\theta\,(1 - 2\mu_E^2) + 3\mu_E \cos\theta] + e^{-2\mu_E\theta} \left[ v_0^2 - \frac{6\mu_E Rg}{4\mu_E^2 + 1} \right]} \tag{37}$$

## 5. Experimental Studies and Predicted Performance

### a) Straight Inclined Chutes of Circular Cross-Section

In order to examine the reliability of the chute flow model developed in the preceding sections, investigations were conducted on the test rig of Fig. 5. Studies were performed using powdered alumina flowing in a clear plastic tube 0.0732 m inside diameter and 2.0 m long.

The coefficient of wall friction $\mu$, was determined by the two methods, firstly by using a shear cell apparatus with samples of the chute material and bulk solid; secondly by the angle of repose method in which the sliding of slugs of material was examined. The latter method enabled both $\mu$ and $k_{EO}$ to be estimated. Owing to electrostatic influence, some difficulty was experienced in determining $\mu$ precisely, since $\mu$ was found to vary during testing. Values of the coefficients $C$ and $\beta_v$ were estimated by examining measured velocity variations along the chute.

Fig. 14 shows a comparison between the computed and experimental results for the chute slopes $\theta = 55°$ and $\theta = 45°$, these angles being measured from the vertical. A test length of 1.8 m was used for the study. Fig. 14(a) shows the velocity variations while Fig. 14(b) shows the stream thickness variations. Movie photography was used to examine the flow and to provide a means for estimating stream velocity. Taking into account the difficulty of determining precisely the various drag coefficients in the chute flow equations the agreement between the predicted and actual results presented in Fig. 14 is considered to be reasonable.

As a further example, the flow through the variable diameter straight inclined chute of Fig. 15 was examined. Fig. 16 shows the variation of the velocity $v$, thickness $H$ and ratio $H/B$ as functions of the displacement $s$. The predicted stream thickness variation generally compares favourably with the experimental values. As indicated, $H/B$ increases in the tapered section but it is important that this ratio should not increase beyond 0.5. For comparison purposes, the predicted performance results for a chute of constant diameter $B = 0.0948$ m are shown by the dotted curves in Fig. 16.

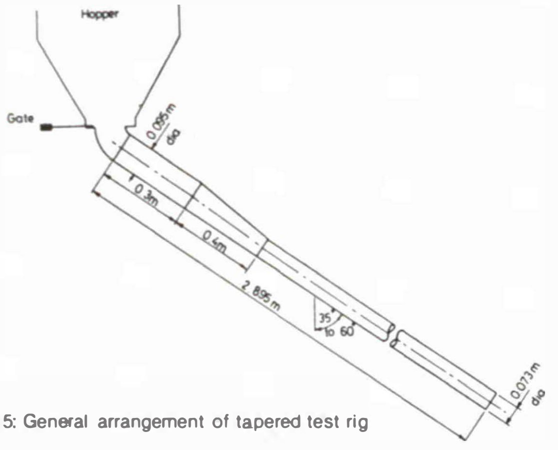


Fig. 15: General arrangement of tapered test rig

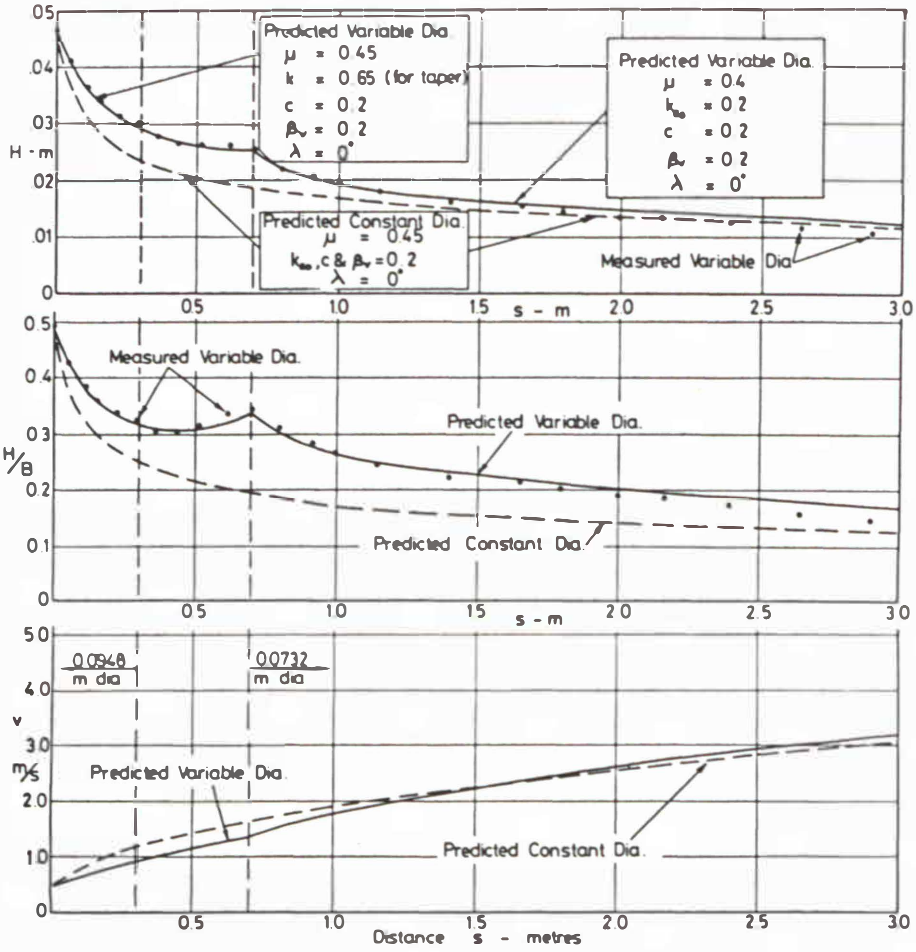


Fig. 16: Comparison of predicted and experimental results for variable diameter straight inclined chute with tapered section.
Material flowing — alumina powder
Chute slope angle — $\theta = 55°$
Chute proportions given in Fig. 15

#### b) Circularly Curved Chutes

Fig. 17 shows a typical set of predicted performance graphs for $H/B$ as functions of angle $\theta$ for a series of circularly

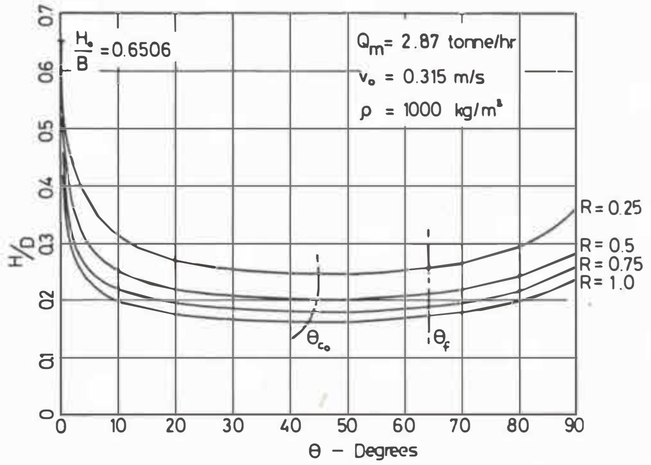


Fig. 17: Predicted performance curves for circularly curved chutes of circular cross-section

curved chutes of circular cross-section. In this case the initial velocity has been taken to be 0.315 m/s and the chute cross-sectional diameter $B = 0.0732$ m. The stream thickness ratio $H/B$ has been determined using equation (25) together with (28) and (29).

The set of curves indicates the influence of chute radius of curvature $R$ on the build-up in stream thickness around the chute. Overall the build-up in thickness is more pronounced in the case of lower radii of curvatures. The thickness also increases with increase in initial velocity.

In deciding the cut-off point for curved chutes, the maximum cut-off angle should never be greater than the limiting slope angle $\theta_f$ which may be approximated by

$$\theta_f = \tan^{-1} \mu_E \text{ at exit.}$$

However, it is desirable, where possible, to terminate the chute at the optimum cut-off angle $\theta_C$ which corresponds to the point where the stream thickness is a minimum and average velocity a maximum. The optimum cut-off angles for the examples shown in Fig. 17 are indicated by the $\theta_{CO}$ curve.

#### c) Chutes of Three Dimensional Geometry

Although this paper has concentrated entirely on chutes of two dimensional geometry, the concepts presented may be readily extended to the three dimensional case. Using the lumped parameter model approach, the equations of motion may be expressed in the most convenient co-ordinate system relevant to the particular chute profile. In practice either cartesian co-ordinates or cylindrical co-ordinates would normally be used. A study of chutes of three-dimensional form has been undertaken by Flottmann [26].

## 6. Conclusions

For efficient and reliable performance, transfer chutes should be designed with a view to obtaining stable 'fast' or accelerated flow conditions. These conditions are met when the thickness of the flowing stream continues to decrease along the chute or, at least, approaches a constant asymptotic value.

The lumped parameter model theory presented in this paper provides an acceptable basis for chute design. The generalized expression for Coulomb frictional and velocity dependent drag is shown to apply to chutes of both rectangular and circular cross-sections.

In view of the tapering stream thickness under accelerated flow in chutes of circular cross-section, it is possible to reduce the chute cross-section diameter, provided that the converging or tapered section of the chute does not introduce a major flow obstruction. In the case of chutes of circular cross-section, the stream thickness build-up due to reducing diameter or flow deceleration should not be permitted to increase beyond the chute cross-sectional radius. That is, the chute should not run more than half full in a deceleration zone.

It is clear that the flow of bulk solids through transfer chutes is, fundamentally, a complex problem and while some substantial contributions have been made, more basic research is needed. In the meantime the design procedures presented in this paper, while embodying several simplifying assumptions, provide a general description of the dynamic conditions governing flow through chutes of various geometrical form with particular emphasis on straight inclined chutes and chutes of constant curvature.

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