bulk solids handling Volume 1, Number 2, May 1981

Analysis of Slip of a Particulate Mass in a Horizontal Pipe

K. C. Wilson, Canada

Das Gleitverhalten einer Feststoffmasse in einem horizontalen Rohr
Le comportement au glissement d'une masse de matières solides dans une tube horizontal
El deslizamiento de una masa sólida en un tubo horizontal

水平パイプ内の粉体の滑り分析

横管内粒体物质滑动的分析

تحليل انزلاق كتلة دقائقية في أنبوب أفقي

Das Gleitverhalten einer Feststoffmasse in einem horizontalen Rohr

Der Druckgradient, der erforderlich ist, um einen Pfropfen von Feststoffteilchen in einem horizontalen Rohr in Bewegung zu setzen, kann von der Pfropfenlänge in mehr als einer Art abhängen. Eine lineare Veränderung, die mit einigen theoretischen Vorhersagen übereinstimmt, impliziert, daß Pfropfenförderung technisch durchführbar ist. Eine progressive Veränderung, die von anderen Analysen vorhergesagt wird, führt in der Praxis zu übermäßig hohem Druck oder vollständiger Rohrverstopfung. Dieser Beitrag schlägt eine allgemeine Formulierung vor, die sowohl den linearen wie den nicht-linearen Fall als spezielle Lösungen einschließt. Es wird gezeigt, daß nur zwei grundlegende Parameter von Bedeutung sind, und zwar eine dimensionslose Größe für die Pfropfenlänge sowie ein Indikator für die fraktionelle Änderung des Flüssigkeits-Druckgradienten in Bezug auf den Logarithmus der Feststoff-Normalspannung. Experimentelle Beweise deuten darauf hin, daß die ortsabhängigen Bedingungen am stromabwärts gelegenen Ende der Feststoffmasse ebenfalls von Bedeutung sind.

Le comportement au glissement d'une masse de matières solides dans un tube horizontal

Le gradient de pression qui est nécessaire pour mettre en mouvement un bouchon de particules solides dans un tube horizontal, peut, dans plus d'une façon, dépendre de la longeur du bouchon. Un changement linéaire qui concorde avec quelques prévisions théoriques, implicite que le transport de bouchon est techniquement réalisable. Un changement progressif, qui est prévu par d'autres analyses, conduit, dans la pratique, à une pression excessivement élevée ou bien à une obstruction totale du tube. Cet exposé propose une formulation générale qui inclue aussi bien le cas linéaire que non-linéaire comme solutions particulières. On montre que seul deux paramètres fondamentaux sont d'importance, à savoir une grandeur sans dimension pour la longueur du bouchon et un indicateur pour la modification fractionnelle du gradient de pression du liquide par rapport au logarithme de la contrainte normale des matières solides. Des preuves expérimentales donnent à penser que les conditions dépendantes du lieu, c'est-à-dire en aval de la masse de matières solides, sont aussi importantes.

El deslizamiento de una masa sólida en un tubo horizontal

La gradiente de la presión que es necesaria para poner en movimiento un cúmulo de partículas sólidas en un tubo horizontal, puede depender de la longitud del cúmulo en más de una especie. Un cambio linear que está de acuerdo a pronósticos teoréticos, implica que el transporte del cúmulo es técnicamente posible. Un cambio progresivo que es anunciado por otros análisis, conduce en la práctica a una excesiva presión y a una completa obstrucción del tubo. La presente contribución propone una expresión general que considera tanto el caso lineal como el no-lineal en soluciones especiales.

Se muestra que sólamente dos parámetros son de importancia, a saber un valor sin dimensión para la longitud del cúmulo y un indicador para el cambio fraccional de la gradiente de la presión del liquido en relación al logaritmo de la tensión normal del sólido. Pruebas experimentales demuestran que las condiciones dependientes del lugar en el extremo de la masa sólida situado flujo abajo, también son importantes.

Summary

The pressure differential required to set a plug of particles in motion along a horizontal pipe may vary with plug length in more than one fashion. A linear variation, which agrees with some theoretical predictions, implies that dense-phase conveying is technically feasible. However, a progressively-increasing variation, which has been predicted by other analyses, leads in practice to excessive pressure or complete line blockage. This paper proposes a generalized formulation that yields both linear and non-linear cases as particular solutions. It is shown that only two basic parameters are involved — a dimensionless measure of plug length and an indicator of the fractional change in fluid pressure gradient with respect to the logarithm of particulate normal stress. Experimental evidence suggests that local conditions near the downstream end of the particulate mass are also important.

Prof. Dr. K.C. Wilson, Department of Civil Engineering, Queen's University at Kingston, Ontario, K7L 3N6, Canada

Nomenclature

- a coefficient in (5)
- b coefficient in (5)
- c volumetric solids fraction
- g gravitational acceleration
- K ratio of radial to axial stress at $z = 0$
- L length of particulate mass, see Fig. 1
- M mass flux (fluid density times superficial velocity)
- p fluid pressure
- r internal radius of pipe
- S coefficient in (4)
- W coefficient indicating effect of granular stress on fluid pressure gradient, see (8)
- x axial co-ordinate, see Fig. 1

z vertical co-ordinate, see Fig. 1
ζ ratio σ/σ_o
η ratio $\bar{\tau}/\mu_s K_o \sigma_o \zeta$
Λ length parameter $2\mu_s K_o L/r$
μ_s coefficient of mechanical sliding friction
ξ ratio x/L
ϱ_e submerged effective density (density of solids minus density of fluid)
σ intergranular normal stress along the pipe axis
σ_r radial granular stress
τ tangential stress between particles and pipe wall

Subscript o refers to conditions at the origin of Fig. 1
Subscript p refers to moving plug with linear pressure drop
Overbars denote average values

1. Introduction

More than one type of behavior can occur when differential fluid pressure is applied across a horizontal length of pipe containing a mass of granular particles. In some circumstances the particulate mass is set in motion as one or more plugs, each completely filling the cross-section of the pipe, and typically having clearly-defined interfaces at both ends. For many cases of this kind the pressure difference required to move a plug is directly proportional to the plug length. This behavior, known as plug flow or dense-phase flow, has been the subject of extensive experimental investigation at Imperial College of Science and Technology, University of London, and it was found by Wilson, Streat and Bantin [5] that the resulting data are in good accord with an analytic model developed at Queen's University at Kingston, Canada.

However, it sometimes happens that dense-phase flow does not occur, and instead the plug appears to jam in the line. This condition is non-linear, with the pressure difference increasing with plug length at an increasing rate, and thus excessively large pressure differences may be required to clear the jam. More than twenty years ago Ede [2] studied this case and concluded that the pressure difference required to clear the blockage was related exponentially to the length of the plug. In the intervening years other attempts at analyzing this non-linear case have been made, but it would appear that little real progress has been achieved either in the mathematical development or in distinguishing the physical mechanisms which would allow prediction of whether linear or non-linear behavior can be expected in any specific instance.

The linear formulation mentioned above has ignored stress-induced deformation of the particulate mass but can take into account a seepage flow of fluid in the interstices between the particles. This seepage sets up drag forces on the particles which act in the same fashion as an additional downstream-directed body force. On the other hand, analyses leading to non-linear solutions have generally ignored the equivalent body force caused by interstitial seepage, and have often ignored the true gravity-associated body force as well. Stresses between the particulate mass and the pipe wall vary along the length of the plug (as opposed to the case dealt with by the linear analysis) but a relation between inter-particulate stress and deformation of the particulate mass is not generally given in explicit formulation.

Recent developments in soil mechanics as shown in Schofield and Wroth [3] have led to some agreement as to the stress-deformation relation; and this should provide a valuable component for a new more generalized analysis. The new analysis, presented below, will also incorporate both the variation of stress with length along the plug (found in the non-linear models) and the effects of seepage flow and the induced equivalent body force (from the linear model mentioned previously).

2. Analysis

As shown on Fig. 1, the x-axis is defined as running horizontally along the pipe centreline, the z-axis is directed vertically upwards and r denotes the pipe radius. The fluid pressure is p, and σ denotes the intergranular normal stress along the pipe axis. Taking the force balance for the contents of the pipe between x and $x + dx$, and dividing by the cross-sectional area gives

$$\frac{dp}{dx} + \frac{d\sigma}{dx} = \frac{2\bar{\tau}}{r} \qquad (1)$$

where τ is tangential stress between the solids and the pipe wall and $\bar{\tau}$ represents an average over the periphery of the pipe.

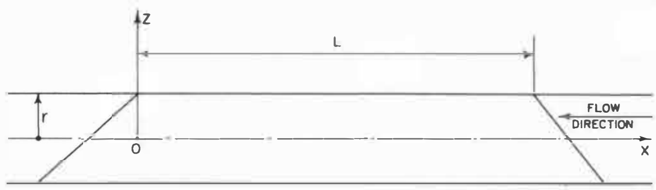

Fig. 1: Definition of quantities

The stress τ can be related to the radial granular stress σ_r which is normal to the pipe wall, since at incipient motion τ equals $\mu_s \sigma_r$, where μ_s is a coefficient of sliding friction. Using K for the ratio of radial to axial stress at $z = 0$, and assuming, with Wilson, Streat and Bantin [5], that σ_r varies with z in accordance with the submerged weight of the solids column; σ_r can be expressed as

$$\sigma_r = K\sigma - \varrho_e g c z \qquad (2)$$

Here ϱ_e is submerged effective density (density of solids minus density of fluid), g is gravitational acceleration, and c is volumetric solids fraction (volume of solids/total volume). The solids fraction c depends on intergranular stress and thus may vary in both the x and z directions. However, for pipes of the size used for conveying, it can be expected that the top-to-bottom variation of c is negligibly small, and c, together with K, will be taken as varying in the x direction only.

At incipient motion of a plug, $\bar{\tau}$ can be evaluated by multiplying (2) by μ_s and averaging over the pipe periphery. It can be shown that the final term of the equation will make no net contribution to the average $\bar{\tau}$ which is simply $\mu_s K\sigma$.

It is convenient at this point to refer the variables to conditions at some particular point and let $c_o K_o$ and σ_o represent the values at the origin of Fig. 1. The position of the origin has been chosen so that here σ_r equals zero at z equals r, and thus, from (2), $\sigma_o = \varrho_e g c_o r/K_o$. Other values of stress can be expressed using the ratio ζ where $\sigma = \zeta\sigma_o$. This gives $d\sigma = \sigma_o d\zeta$ and, for incipient motion, $\bar{\tau} = \mu_s K_o \sigma_o \zeta (K/K_o)$. If the point of incipient motion has not been reached, $\bar{\tau}$ will have a smaller value and this effect, together with the ratio (K/K_o), is incorporated into a variable η, defined so that $\bar{\tau} = \mu_s K_o \sigma_o \eta \zeta$. Substitution into (1) yields

$$\frac{dp}{dx} + \sigma_o \frac{d\zeta}{dx} = 2\mu_s \frac{K_o \sigma_o}{r} \eta \zeta \qquad (3)$$

The pressure gradient through a granular medium can be expressed in terms of the mass flux M (fluid density times superficial velocity) and the solids fraction c. The simplest relation, the Kozeny-Karman equation which is cited in standard works such as Coulson and Richardson[1], may be written

$$\frac{dp}{dx} = SM \frac{c^2}{(1-c)^3} \qquad (4)$$

Here S incorporates a numerical coefficient times the kinematic viscosity of the fluid and the square of the specific surface (which, in turn, is a function of grain size). This equation applies when the interstitial flow is laminar. Equations with higher powers of M are available to deal with turbulent flow, but would appear to offer no advantage in the present application, since it is expected that the interstitial flow will be laminar in cases where blockage may occur.

The solids fraction is itself a function of intergranular stress, and following Schofield and Wroth[3] it is expected that the specific volume $1/c$ will decrease linearly with the logarithm of intergranular pressure. Strictly speaking, the intergranular pressure is the mean of the three orthogonal normal stresses but, in practice, tests are run for unidimensional compression where the axial stress component, equivalent to σ, is taken as the independent variable. The results of these tests give the coefficients (a and b) in the expression

$$1/c = a - b \ln \sigma \qquad (5)$$

On substituting $\sigma_o\zeta$ for σ and introducing the reference concentration c_o, this equation can be rearranged to give

$$1/c = 1/c_o - b \ln \zeta \qquad (6)$$

Equation (6) could be used directly to evaluate the function of c found in (4), but the complexity of the resulting expression tends to obscure the relationship rather than elucidating it. An alternate approach is to express the function as a Taylor series in $\ln\zeta$, with the coefficients evaluated at $\zeta = 1$. After rearrangement, and neglecting higher order terms, this yields

$$\frac{c^2}{(1-c)^3} \simeq \frac{c_o^2}{(1-c_o)^3}(1 + W \ln \zeta) \qquad (7)$$

Here the coefficient W is given by

$$W = \frac{bc_o(2 + c_o)}{(1-c_o)} \qquad (8)$$

and can readily be evaluated from the results of bench-top tests. It can be shown that the neglect of higher order terms on the right hand side of (7) must lead to an under-estimate, but calculations with the worst combinations of c_o and b which have been observed in tests performed at Warren Spring Laboratory[4] indicate that this effect would only be important if ζ should exceed 10. On the other hand, if ζ is less than about 1.2 a further simplification may be made by replacing $\ln\zeta$ by $\zeta - 1$.

Analysis of (3) can now be undertaken, and it is convenient to begin with the simplest case — a sliding plug with no change in σ along the x axis (so that $d\sigma/dx$ and $d\zeta/dx$ equal zero). Conditions at the downstream end of the plug require that ζ equals unity at that point, and hence throughout the length (from which $\ln\zeta = 0$). Likewise K/K_o, and hence η, will be unity so that (3), with substitution of (4) and (7), becomes

$$\left[\frac{dp}{dx}\right]_p = S M_p \frac{c_o^2}{(1-c_o)^3} = 2\mu_s K_o \frac{\sigma_o}{r} \qquad (9)$$

Here the subscript p indicates that the pressure gradient and mass flux which appear here apply only to the case of the moving plug. On substituting for σ_o, it is seen that the expression for the plug-flow pressure gradient can also be written $2\mu_s \varrho_e g c_o$, in accord with earlier work on flow of this type by Wilson, Streat and Bantin[5].

Equation (9) can now be used to eliminate the quantity $Sc_o^2/(1-c_o)^3$ in the general case where M will not coincide with M_p; and (3) thus becomes

$$\frac{M}{M_p} \cdot 2\mu_s K_o \frac{\sigma_o}{r}(1 + W \ln \zeta) + \sigma_o \frac{d\zeta}{dx} = 2\mu_s K_o \frac{\sigma_o}{r} \eta\zeta \qquad (10)$$

It is convenient to introduce two quantities based on the length L of the pipe occupied by solids. These are the variable $\xi = x/L$ (from which $dx = L\,d\xi$) and the dimensionless length parameter $\Lambda = 2\mu_s K_o L/r$. Substitution and multiplication by L/σ gives

$$\frac{d\zeta}{d\xi} = \Lambda\left[\eta\zeta - \frac{M}{M_p}(1 + W \ln \zeta)\right] \qquad (11)$$

In this basic dimensionless equation, ζ and η are dependent variables and the parameters W and Λ, which have already been discussed, can be evaluated without difficulty.

The remaining parameter, the ratio M/M_p, can be related to the mean values of the dependent variables. These means, obtained by integrating with respect to ξ within the limits zero and one, are denoted by a raised bar; for example $\overline{\eta\zeta} = \int_0^1 \eta\zeta\, d\xi$. In the case of the slope $d\zeta/d\xi$, the mean is simply the difference of the values of ζ at the two ends of the granular mass. Moreover, as $\zeta = 1$ at both the upstream and downstream end, this difference will equal zero, so that, for mean values, (11) reduces to

$$\overline{\eta\zeta} = \frac{M}{M_p}(1 + W \overline{\ln\zeta}) \qquad (12)$$

If the expression for M/M_p from (12) is substituted into (11), it can be seen that the only externally imposed parameters are W and Λ, but that the equation is of mixed integral-differential type. The solution of such equations generally presents formidable difficulties, although numerical evaluations may be made by means of variational techniques. There would appear to be no likelihood of a simple closed form solution, certainly not the exponential proposed by Ede[2].

3. Remarks

No additional formal analysis of the equations will be attempted here, but it may be useful to examine any limits in the ranges of the variables and to indicate, in a qualitative sense, the effect of variations in the parameters. As defined, the parameters W and Λ are inherently non-negative, and the same applies to M and M_p. The variable ζ is equal to unity throughout the length L for the limiting case of plug flow which was dealt with in connection with (9), and in that case η and $\eta\,\zeta$ both equal unity, whereas ln ζ equals zero. For the non-linear case where blockage has formed, ζ will exceed unity along at least part of the length, but the limiting conditions will apply locally at the downstream end. At this point, where $\xi = 0$, (11) simplifies to

$$\left[\frac{d\zeta}{d\xi}\right]_{\xi=0} = \Lambda\left[1 - \frac{M}{M_p}\right] \tag{13}$$

It can be seen that a positive slope $d\zeta/d\xi$ will be required to raise ζ from its unit value at the downstream end to the higher values required at larger ξ, and for this to be the case (13) indicates that M/M_p must be less than unity by some finite, but not necessarily large, amount.

In the non-linear case the pressure drop, Δp, can be higher than the value for a sliding plug, which is given by the product of the length L and the plug-flow gradient of (9). A general expression for $\Delta p/L$ can be obtained by integration of (3), recalling that it has been shown that $d\zeta/dx$ will make no net contribution. After rearrangement, this yields

$$\frac{\Delta p}{L} = \left[\frac{dp}{dx}\right]_p \overline{\eta\,\zeta} = 2\,\mu_s\, p_e\, g c_o\, \overline{\eta\,\zeta} \tag{14}$$

For a block of given length, it is possible to set up a pressure difference Δp and to increase this until a maximum is reached just before motion begins. It may be that the first motion will be that of the block as a whole, as in the plug-flow case, but if the stress distribution is favourable for blockage this may not occur directly. Instead, Δp will reach an upper limit when the local pressure gradient at the downstream end of the granular mass becomes sufficient to set the particles there in motion, leading to a progressive unravelling of the mass of grains and, eventually, the movement of the remaining block. The motion of particles at the downstream end requires a fluid mass flux equal to M_p, that is, a ratio M/M_p of unity. As shown previously, M/M_p cannot, strictly speaking, reach unity and thus the value of $\Delta p/L$ associated with M/M_p of unity must be considered as an upper limit to be approached, rather than reached. From (12) and (14), it follows that this limit is given by

$$\frac{\Delta p}{L} < \left[\frac{dp}{dx}\right]_p (1 + W\,\overline{\ln\zeta}) \tag{15}$$

Equation (15) shows that the parameter W has a direct effect on the maximum pressure drop which can be sustained by a granular block. For the limiting case of W approaching zero (which implies that the slope b of (5) approaches zero), it appears that $\Delta p/L$ cannot exceed $(dp/dx)_p$. The effect of the dimensionless length Λ cannot be seen immediately from (15), but (11) shows that $d\zeta/d\xi$ is directly proportional to Λ; and thus it can be expected that the mean of ζ, and hence $\overline{\ln\zeta}$ and $\Delta p/L$, will increase with increasing Λ.

The effect of Λ which has just been noted is in general accord with that given by experience for the length-to-radius ratio of a plug. In fact, expected values of μ_s and K_o put the product $2\,\mu_s K_o$ reasonably close to unity so that Λ is not far from L/r.

Numerical values of the parameter W have been calculated from data provided by Warren Spring Laboratory from their tests of a wide variety of materials done for Imperial Chemical Industries. The W values range from 0.020 to 0.177, and it is believed that almost all materials which may be encountered in practice will have W below 0.20. For such values it can be expected that the upper limit of $\Delta p/L$ will increase monotonously with increasing W.

In pilot plant testing, and *a fortiori* in industrial experience, it is very difficult to obtain measurement of $\Delta p/L$ for cases of blockage, where this quantity considerably exceeds the plug-flow gradient $(dp/dx)_p$. In these cases the paucity of measurements is usually supplemented by remarks as to jamming tendencies and the impracticability of conveying the material being tested. The results from Warren Spring Laboratory suggest that the magnitude of W is helpful but not sufficient in estimating the occurrence of such conditions. Specifically, totally unsatisfactory performance was found for materials with W as low as 0.046, but other materials with larger W (as high as 0.177) could be conveyed without undue difficulty.

It is evident that other effects must also be considered to obtain a complete picture of the blockage phenomenon. One possibility is that the main bulk of the block may display properties (for example, effects of non-uniform particle size or very low permeability) which have not been included in the parameters analyzed above. To an extent, this problem can be overcome by improved bench-top testing, perhaps by modifying the compressive testing apparatus to permit direct measures of permeability (instead of $1/c$) at various granular pressures, thus obtaining a more direct determination of W.

It is believed, however, that the major problems of modelling do not lie in the properties of the main granular block, but at the downstream end. It has been found in the laboratory that lateral fluid loss (for instance at a leak or at certain types of exit configuration) can trigger extensive blockages. It can now be seen that the ratio M/M_p will be reduced at, and downstream of, the lateral loss, which will cause an increase in $d\zeta/dx$ (see (11), (13)) and hence in lnζ and $\Delta p/L$. It is expected that other local conditions at the downstream end (possibly arising from cohesion or from deviation from (5) at low granular stress) can act in a similar way to trigger blockage. Increased understanding of such local phenomena in particulate materials may sometimes be obtained by deaeration (consolidation) tests and by observing behavior in tilting tubes, where, as shown in Wilson, Streat and Bantin [5], the axial component of gravitational attraction is closely analogous to the force produced by the seepage flow in a prototype pipeline situation.

4. Conclusions

A new, generalized, analysis has been carried out on the slip of a particulate mass occupying a horizontal pipeline. The analysis includes treatment of interstitial seepage and the equivalent body force which this produces. The equivalent body force has been related to the deformation produced by interparticulate stress, and the effect of the normal stress of the grains against the pipe wall has been incorporated into the analysis.

On simplifying the resulting equation and reducing it to dimensionless form, it was found that the effects of plug length, pipe radius and particle and fluid properties can be reduced to two independent ratios, denoted as Λ and W. The ratio Λ is a dimensionless measure of plug length, and W

indicates the fractional change in fluid pressure gradient (or equivalent body force) with respect to the logarithm of particulate normal stress.

For a zero value of W (i.e. an *incompressible* granular matrix for which permeability is not influenced by interparticulate stress) the analysis gave only a linear solution, with particulate stress not dependent on plug length and a constant value of fluid pressure gradient.

For larger values of W this linear case remained as one possible solution, but it would appear that one or more non-linear solutions are also possible, at least for sufficiently large values of the relative plug length Λ. The likelihood of a non-linear solution, with resulting excessive pressure drop, should increase with increases in both W and Λ, but the experimental evidence shows that this does not give the complete picture.

Specifically, it is found both in the laboratory and as a result of the analysis that the shift from the linear to the non-linear case (in practice the difference between dense-phase flow and blockage) is sensitive to local conditions at the downstream end of a mass of particles. It is doubtful that a mathematical model alone can deal with such localized anomalies in the behavior of granular materials, and thus it is believed that a combination of bench-top testing and mathematical modelling is needed to resolve the problem of transition from linear to non-linear behavior.

Acknowledgements

This work was made possible by a sabbatical leave granted by Queen's University, Canada, and spent, in part, at the Warren Spring Laboratory, UK.

References

[1] Coulson, J. M., and Richardson, J. F., Chemical Engineering, Vol. 2, Pergamon, London, 1968

[2] Ede, A. N., "The Resistance of Concrete Pumped through Pipelines", Magazine of Concrete Research, pp. 129—140, November 1957

[3] Schofield, A. N., and Wroth, C. P., "Critical State Soil Mechanics", McGraw-Hill, London, 1968

[4] Wilson, K. C., "Analytic Modelling of Energy Consumption in Solids Pipelining", Proc. Pneumotransport 5, BHRA Fluid Eng'g, London, 1980.

[5] Wilson, K. C., Streat, M., and Bantin, R. A., "Slip Model Correlation of Dense Two-Phase Flow", Proc. Hydrotransport 2, BHRA Fluid Eng'g, Coventry, 1972