bulk solids handling Volume 1, Number 2, May 1981

Optimal Design of Continuous Conveyors

A. W. Roberts, J. W. Hayes and O. J. Scott, Australia

Die optimale Auslegung kontinuierlicher Fördermittel
La conception optimale de moyens de transport continus
El diseño óptimo de transportadores de material continuos
連続コンベアの最適設計
连续输送机的最佳设计
التصميم المثالي للناقلات التي تعمل بصورة مستمرة

Die optimale Auslegung kontinuierlicher Fördermittel

Dieser Beitrag stellt ein Verfahren für die optimale Auslegung für kontinuierliche Schüttgut-Fördermittel vor. Das Problem liegt in der Aufstellung von Kostenfunktionen, die die Leistungsfähigkeit von Anlagen mit verschiedenen Kostenfaktoren korreliert. Unter Zugrundelegung von allgemeinen Grundlagen der ingenieurtechnischen Kostenanalyse wird gezeigt, daß Kostenfunktionen abgeleitet werden können, die die Energiekosten und die äquivalenten jährlichen Gerätekosten berücksichtigen. Die letzteren müssen Faktoren wie Lebensdauer von Anlagen und deren Komponenten, Wiederverkaufswert, Steuern sowie Geldrücklaufrate enthalten. Inflation sowie die jährlich ansteigenden Energiekosten werden in dem Modell berücksichtigt. Verschiedene Optimierungsmethoden werden behandelt und die optimale Auslegung eines Fördersystems wird anhand eines Beispiels beschrieben. Angaben über Konstruktionsvergleiche für verschiedene Fördermittel werden gemacht.

La conception optimale de moyens de transport continus

Cet expose présente un procédé pour concevoir de façon optimale les moyens d'acheminement continus de matières en vrac. Le problème est d'établir des fonctions de dépenses qui fassent correspondre la productivité de l'installation aux différents facteurs de dépenses. On montre à partir des principes généraux de l'analyse des dépenses de l'ingénierie technique que des fonctions de dépenses qui prennent en considération les frais d'énergie et les frais d'équipement annuels équivalents, peuvent être déterminées. Ces derniers doivent tenir compte de facteurs tels que: longévité de l'installation et de ses composantes, valeur de la revente, impôts et dividendes L'inflation tout comme les frais d'énergie qui augmentent annuellement sont aussi pris en considération dans le modèle. On s'intéresse à différentes méthodes optimalisantes et on décrit la conception optimale d'un système de transport à l'aide d'un exemple. On donne également des indications sur les comparaisons de construction de différents moyens de transport.

El diseño óptimo de transportadores de material continuos

La presente contribución presenta un concepto para un diseño óptimo de transportadores continuos para materiales a granel. El problema está en establecer funciones de costo que integren el rendimiento de las plantas con los diferentes factores de costo. Basándose en principios generales de los análisis de la ingeniería económica se muestra que se pueden derivar funciones de costo que consideran el costo de energía y el costo anual equivalente de equipos. Los últimamente nombrados tienen que contener factores como ser: duración de las plantas y sus componentes, valor de reventa, impuestos y cuotas de reflujo. Tanto la inflación como el costo de energía que anualmente suben, son considerados es este modelo.

Diferentes métodos de optimización son tratados. El diseño óptimo de un sistema de transporte se describe por medio de un ejemplo y se dan datos sobre comparaciones en la construcción de diferentes medios de tranporte.

Summary

This paper presents a procedure for the optimum design of continuous conveyors for bulk solids handling. The problem concerns the establishment of cost or objective functions which integrate the performance characteristics with the various cost factors involved. Using the general principles of engineering economic analysis, it is shown that cost functions may be derived which take into account the energy costs and annual equivalent cost of equipment. The latter requires consideration of such factors as equipment or component life, salvage value, taxation rates and rates of return. The effects of inflation and variations in the annual differential escalation in the energy component costs are included in the model. Various optimisation techniques are discussed and, by way of example, the optimum design of belt conveyor systems is described. Design information is presented for comparisons to be made between different modes of conveying.

1. Introduction

As in any engineering design exercise, the design and selection of conveyors and handling equipment for a particular process or system involves the consideration of a number of alternative solutions. The overall or global problem requires comparisons to be made between different types of equipment and modes of transport with economic considerations playing a major role in the final decision making. When different modes of conveying and transportation are compared, such as belts, buckets, pipelines, rail and road, the variations in costs may differ by several orders of magnitude. Even when one mode of conveying, such as belt conveying, is examined for a particular installation, within the range of possible combinations of conveyor size, speed and geometrical layout, there can be considerable variations in the overall costs. For the reasons stated it is particularly

Prof. Dr. A.W. Roberts, J.W. Hayes and O.J. Scott,
Department of Mechanical Engineering, The University of Newcastle,
N.S.W. 2308, Australia

Based on a paper presented at Powder Europa 80 Conference, Wiesbaden, West Germany, January 1980

important that the conditions for optimum performance of particular types of conveyors and handling equipment be established. Furthermore, in view of the heavy dependence by industry on bulk materials handling operations, any increase in efficiency, even if only small, can lead to substantial cost savings.

In cases where particular types of conveyors or elevators are specified, consideration needs to be given in the design process to the many combinations of variables which may satisfy the required performance criteria. As a consequence it is necessary that optimum solutions be sought with respect to cost or objective functions which take into account the relevant factors contributing to the overall cost of the operation. These solutions are subject to the constraints imposed by design, manufacturing and operational limitations. The optimum solutions obtained by analysis and numerical techniques may be implemented directly or used as a *yardstick* against which the actual conveyor performance can be measured.

This paper describes a procedure, based on engineering economic analysis, for the optimum design of continuous conveyors. The general concepts presented are based on those established firstly by Roberts and Charlton [1] and further developed by Roberts, Hayes and Scott [2, 3, 4]. Cost or objective functions are derived which take into account energy costs and the annual equivalent capital cost of the equipment comprising the conveyor or system of conveyors. In the latter case such factors as the equipment life, salvage values, taxation rates and rates of return are included in the design optimization model. The effects of inflation are also included and the influence of reliability and maintainability in the decision making process is discussed.

To illustrate the application of the procedure, the design and selection of belt conveyors are discussed in some detail and mention is made of the application to enclosed screw or auger conveyors and bucket elevators. This information provides a basis for handling systems design where comparisons need to be made between different modes of conveying.

2. The Generalised Conveyor Design Problem

Consider the problem of conveying a bulk material from one point to another by some form of continuous conveyor or system of conveyors. A range of possibilities exists, as illustrated in Fig. 1. Where it is required to elevate a bulk material through a prescribed height of lift and discharge it at a specified rate, by way of example, the use of a belt conveyor, pneumatic conveyor, bucket elevator or enclosed screw or auger conveyor as shown in Figs. 1(a), (b), (c) and (d) respectively, may be contemplated. The choice will depend, to a large extent, on the characteristics of the bulk material and the requirements of the particular installation. Where it is required to move a bulk material over some distance as well as elevating it through a specified height, it is possible to consider, for example, either a single belt conveyor as in Fig. 1(a) or a combination of conveyors such as the horizontal belt conveyor and bucket elevator as in Fig. 1(e). For long distance transportation by belt conveyor, a multiple conveyor system as in Fig. 1(f) will need to be considered. In such cases it is necessary to determine, on economical as well as other grounds, bearing in mind the overall system constraints, the most desirable number of individual conveyors and their individual lengths.

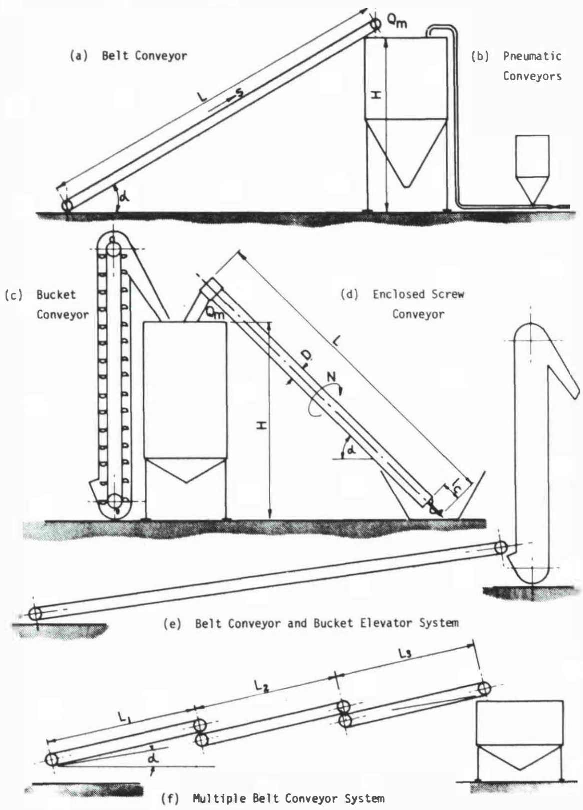

Fig. 1: Typical conveyor configuration

2.1 Performance Characteristics

The design of a conveyor or system of conveyors depends on a knowledge of the relevant performance characteristics. Of particular importance are the relationships for throughput and power. While the design and operating features of the various types of mechanical conveyors differ widely, the basic performance characteristics, in functional form, are quite similar. For a given bulk material, the general form of the relationships for power and throughput is:

1. Power (kW)

$$P_M = f_1(x_1, x_2, \ldots, x_n, \varrho_m, L, s, \alpha) \quad (1)$$

2. Throughput (kg/s or t/h)

$$Q_m = f_2(x_1, x_2, \ldots, x_n, \varrho_m, s, \alpha) \quad (2)$$

In the above relationships (x_1, x_2, ... x_n) are geometrical design variables applicable to the particular conveyor, ϱ_m is the bulk density of the material being conveyed, L is the overall conveyor length, s is the conveyor speed and α is the angle of elevation.

The geometrical variables are those that express the carrying capacity (and power) in terms of a unit conveyor length. For example, in the case of a belt conveyor, they include the belt width, number of plies, idler configuration (number of rollers and troughing angles) and idler spacing. For a screw conveyor they include the screw diameter, pitch, core diameter, choke length and casing clearance. For a bucket elevator they include the belt width, bucket capacity and bucket spacing.

In the case of multiple length conveyors the overall length will be given by

$$L = \sum_{i=1}^{N} \ell_i \tag{3}$$

where

ℓ_i = Length of i th conveyor
N = Number of conveyors involved.

For conveyors used to elevate bulk materials, the overall efficiency is of importance in the performance assessment. The overall efficiency relates the theoretical power to elevate a bulk material in the absence of friction to the actual power. That is

$$\eta_0 = \frac{Q_m g L \sin\alpha}{3600 P_M} \tag{4}$$

where

Q_m = t/h

In addition to the performance equations (1), (2) and (4), it is necessary to establish a relationship which takes into account the over-riding geometrical requirements governing the conveying distance and height of lift. That is, a relationship is needed which expresses the effective height of lift $H(m)$ as a function of conveyor length and angle of elevation α. Knowing the required height of lift, an additional height allowance must be made to permit the gravity flow of the bulk material from the outlet of the conveyor through some discharge device such as a transfer chute.

In general terms

$$H = f_3(L, \alpha) \tag{5}$$

By way of example, an appropriate function for the bucket elevator and screw conveyor of Figs. 1(c) and (d) respectively is

$$H = \frac{L \sin\alpha}{1 + C_H \sin\alpha} \quad \text{for } 0 < \alpha \leq \frac{\pi}{2} \tag{6}$$

where

C_H = Coefficient based on conveyor geometry

2.2 Design Constraints

In addition to the need to satisfy the performance and geometrical requirements, the design analysis must take into account certain additional constraints. These fall into two main groups:

1. Functional Constraints

These govern the need for the conveyor components to be designed for strength, safety and reliability. The actual magnitude of the conveyor geometrical variables may often be dictated by the strength and life characteristics of the component materials. For example the number of plies p in a conveyor belt, as well as depending on the belt width B and maximum belt tension F_{max}, also depends on the safe working stress of the belt material σ_b.

That is

$$p = f_4(\sigma_b, B, F_{max}) \tag{7}$$

2. Constraints on System Variables

Practical considerations usually create the need to constrain the conveyor variables so that they lie within fixed limits. For instance the upper limit on belt conveyor speed is about 6 m/s, this limit being dictated by the need to ensure efficient tracking and to minimize component wear. Belt widths are limited by manufacturing capabilities. The rotational speeds of screw conveyors have upper and lower limits which are dictated by their dynamic performance characteristics. Hence in general terms we can write

$$\begin{array}{ccccc} x_{1l} & \leq & x_1 & \leq & x_{1u} \\ \text{—} & \text{—} & \text{—} & \text{—} & \text{—} \\ x_{nl} & \leq & x_n & \leq & x_{nu} \\ s_l & \leq & s & \leq & s_u \\ \alpha_l & \leq & \alpha & \leq & \alpha_u \end{array} \tag{8}$$

2.3 Design Solution

The design of a conveyor to satisfy the specified performance requirements will involve the consideration of a number of alternative solutions. In theory, computations based on the relationships of (1) and (2) together with the constraints of the type given by (5), (7) and (8) will yield a large number (in fact an infinite number) of solutions in the *solution space*, all of which meet the required performance conditions. A decision needs to be made as to which of the possible solutions is the most appropriate. For this reason additional criteria need to be laid down to aid the decision making process. Such criteria, inevitably, must take account of the need for efficient and economic operation.

By establishing appropriate cost or objective functions it is possible to obtain optimal solutions to conveyor design problems. Such objective functions need to be derived on the basis of detailed economic considerations, as outlined in subsequent sections of this paper. Objective functions obtained in this way will have the functional form

$$I = I(x_1, x_2, \ldots, x_n, \varrho_m, s, L, \alpha) \tag{9}$$

The objective is to determine the system variables (x_1, $x_2, \ldots x_n$, s, L, α) that minimize I subject to the required performance condition or throughput expressed by (2). The solution must take account of the various system constraints such as those given by equations (3), (5), (7) and (8).

In general terms the design analysis is transformed into a constrained, non-linear optimization problem. While several known computing algorithms for this class of problem exist [5], two have been found to be effective for the solutions of conveyor optimization problems. These are the constrained Fletcher-Powell (Conmin) algorithm which has been adapted by Khaw [6] for the solution of screw conveyor design problems and the Box (complex) algorithm which was also examined by Khaw and more recently by Lim [8] for belt conveyor problems. A modification of the complex algorithm based on reference [7] has also been examined by Lim for discrete valued variables optimization problems which are more representative of actual belt conveyor design problems.

While optimum solutions may be found in this way, it is important to note that the optimization algorithm is not going to be the final answer in all cases. At least perturbations of the solution about the optimum will show how sensitive the operating costs are to variations in the performance variables. If the independent variables are few in number and the general range of these variables for a feasible solution is known, then it is often easier to determine the optimum solution by repeated computation of the cost function I for the selected range of variables. This procedure may be preferable where, as in many cases, the variables need to be discrete values or integers such as belt width and the number of plies comprising the belt.

3. Economic Considerations

The costs incurred in a conveyor system may be divided into two categories:

1. Operating costs
2. Capital costs.

Any form of economic evaluation must express both types of cost in some common measure. By specifying a rate of return required on the capital funds employed, costs may be expressed as present equivalent costs or as annual equivalent costs over the life of the system.

Depending on the accuracy required, the analysis may be on the basis of cash flows:

1. Before tax
2. After tax without considering inflation
3. After tax considering inflation.

Only the last of these will be described as the other two are readily derived from it.

If all costs are expressed as the inflated dollar expenses expected to be incurred at the time they occur, then for capital items not needing replacement during the life of the system, n, the present equivalent of these capital costs, PEC, is given by [12]

$$PEC = \frac{A - V\left(\frac{p}{f_n}\right)^{i_f} - t(PED)}{1 - t} \quad (10)$$

where

A = First cost of the item
V = Salvage value
t = Tax rate
PED = Present equivalent of depreciation
i_f = Inflation modified rate of return

$$\left(\frac{p}{f}\right)_n^{i_f} = \frac{1}{(1+i_f)^n} = \text{Present equivalent factor.}$$

For any company maintaining a constant proportion of debt capital, r_d, then i_f is given by

$$i_f = (1-t)\, r_d\, i_d + (1-r_d)\,[(1+r)(1+i_e) - 1] \quad (11)$$

where

i_d = Interest rate on debt
i_e = After tax return required on equity funds with zero inflation rate.

The annual equivalent cost, in year zero dollars, may be obtained by multiplying the present equivalent by the capital recovery factor,

$$\left(\frac{a}{p}\right)_n^{i_f^o} = \frac{i_f^o(1+i_f^o)^n}{(1+i_f^o)^n - 1}$$

that is

$$AEC = PEC\left(\frac{a}{p}\right)_n^{i_f^o} \quad (12)$$

The factor i_f^o which expresses the time value of money when all cash flows are expressed in constant year zero dollars rather than inflated dollars is given by:

$$i_f^o = \frac{(1-t)\, r_d\, i_d - r\, r_d}{(1+r)} + (1-r_d)\, i_e \quad (13)$$

For any item which may require replacement during the life of the conveyor system, the values of A, V and PED in equation (10) are modified to include the present equivalents of the original and all replacement items required.

By expressing salvage values and depreciations as a fraction of first cost A, the annual equivalent cost of any item can be expressed as a fraction of its first cost and hence as a function of the operating and geometrical variables of the system. By expressing all costs in this form and summing, the total annual equivalent cost is obtained.

4. Optimization of Belt Conveyors — General Remarks

In order to illustrate the integration of economic analysis into the optimum design of conveyors, consider the problem of designing a typical belt conveyor installation as shown in Fig. 1(a) or multiple conveyor system of the type shown in Fig. 1(f). The conveyor is required to transport a bulk material of density ϱ_m (kg/m³) at a rate of Q_m (kg/s) over a distance L and height of lift H.

The relevant geometrical design variables are

$x_1 = B$ = Belt width
$x_2 = p$ = Number of plies (Note: p must be an integer)
$x_3 = \beta$ = Idler troughing angle for two roller or three roller idler configuration
$x_4 = \lambda$ = Idler contact perimeter ratio for nominated idler configuration
$x_5 = a_o$ = Idler spacing or carrying side (m)
$x_6 = a_u$ = Idler spacing on return side (m)
$x_7 = \ell$ = Length of individual conveyors in multiple conveyor system (m)
Also $v = s$ = Belt velocity (m/s)

4.1 Performance Characteristics

While the general design procedures for belt conveyors are well documented, for the purpose of the present discussion the essential equations given in references [3] and [4] are summarised below:

Throughput Q_m is given by:

$$Q_m = \varrho_m A\, v \cos\alpha \quad (14)$$

where

$$A = U b_2 \quad (15)$$

U = Non-dimensional cross-sectional area shape factor

Normally the angles of incliniation α are low enough for $\cos\alpha \cong 1$ in equation (14).

By way of example, the shape factors for two and three roller idler systems illustrated in Figure 2(a) and (b) respectively are given by

$$U_2 = \frac{\sin 2\beta}{8} + \frac{\tan\delta}{12}(\cos 2\beta + 1) \quad (16)$$

$$U_3 = \frac{1}{(1+2\lambda)^2}\left\{\lambda\sin\beta + \frac{\lambda^2}{2}\sin 2\beta + \frac{\tan\delta}{6}\left[1 + 4\lambda\cos\beta + 2\lambda^2(1+\cos 2\beta)\right]\right\} \quad (17)$$

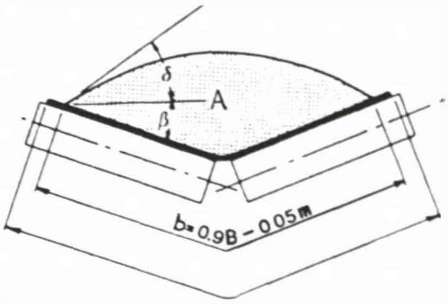

Fig. 2a: Two idler configuration

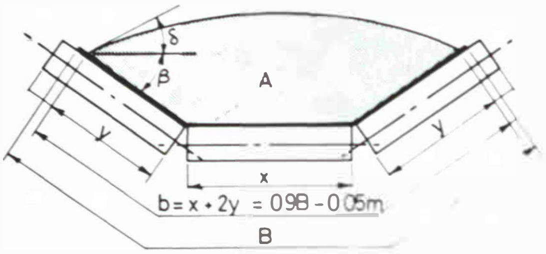

Fig. 2b: Three idler configuration

where

$$\lambda = y/x$$

In equation (15) b is the "wetted" or contact perimeter. The belt width, allowing for edge effects, is

$$B = 1.11\,b + 0.056\,(\text{m}) \qquad (18)$$

4.2 Conveyor Power

In simplified terms the total resistance F_U of a conveyor belt is

$$F_U = C\,[F_{H_1} + F_{H_2}] + F_{st} + F_{s_1} + F_{s_2}\,(\text{kN}) \qquad (19)$$

where

F_{H_1} = Empty belt frictional resistance
F_{H_2} = Load frictional resistance
F_{st} = Slope resistance
F_{s_1}, F_{s_2} = Special resistances
C = Factor to allow for secondary resistances such as those due to accelerating the material onto the belt.

The factor C is given in reference [9]. Alternatively it may be expressed as:

$$C = 0.85 + 13.31\,L^{-0.576} \text{ for } 10 < L < 1500\text{ m}$$
$$C = 1.025 \text{ for } 1500 < L < 5000\text{ m} \qquad (20)$$

The required motor power is

$$P_M = \frac{F_U v}{\eta} \qquad (21)$$

where η = Drive efficiency

4.3 Belt Design

Based on the simplified drive analysis the wrap factor C_w for a given angle of wrap θ (radians) on the driving pulley(s) may be approximated by

$$C_w = \frac{1}{e^{\mu\theta} - 1} \qquad (22)$$

where μ = Friction coefficient between the belt and pulley

The slack side tension is

$$F_2 = C_w F_U \qquad (23)$$

and tight side tension is

$$F_1 = F_U + F_2 \qquad (24)$$

The maximum belt tension will be the larger of the values of F_1 and that computed from the conveyor layout where it is necessary to limit the belt sag between the idlers.

For an assumed belt type, the allowable stress is given by σ_b (kN/m, ply). Thus for a given belt width B the required number of plies is given by

$$p = \frac{F_{max}}{B\,\sigma_b} \qquad (25)$$

where p = Integer value

4.4 Economic Considerations

For the belt conveyor, the specific items to be considered are

a) Operating Costs:

— Energy
— Repairs and maintenance
— Labour

b) Capital Costs:

— Drive system, motor, speed reducers
— Belt
— Idlers
— Structure
— Transfer stations (in multi-conveyor systems)

1. Energy Costs

The annual energy cost may be calculated from the annual energy consumption. That is

$$I_1 = k_1\,e_c\,P \qquad (26)$$

where

e_c = Annual hours of operation times the unit cost of energy
k_1 = Annual equivalent energy cost coefficient taking into account inflation and annual escalation rate of energy costs

It is worth noting that a considerable amount of energy is expended in overcoming frictional resistance. This will vary with the speed and design features of the particular conveyor and the bulk density of the material.

In some studies it may be desirable to allow for the possibility of energy costs rising more rapidly than the general inflation rate. If

r = General inflation rate
r_e = Annual escalation rate of energy costs
e_{co} = Energy cost at time zero
e_{cz} = Energy cost in year z

then:

$$e_{cz} = e_{co}(1 + r_e)^z \quad (27)$$

The present equivalent of energy costs over the life of the system n is given by

$$PEC = \sum_{z=1}^{n} e_{co}\left\{\frac{1 + r_e}{1 + i_f}\right\}^z \quad (28)$$

The annual equivalent in year zero dollars is obtained by multiplying by the capital recovery factor

$$\left(\frac{a}{p}\right)_n^{i_f^o} = \frac{i_f^o(1 + i_f^o)^n}{(1 + i_f^o)^n - 1}$$

that is

$$\text{AEC (energy)} = e_{co}\left(\frac{a}{p}\right)_n^{i_f^o} \sum_{z=1}^{n}\left\{\frac{1 + r_e}{1 + i_f}\right\}^z \quad (29)$$

Thus k_1 in equation (26) becomes

$$k_1 = \left(\frac{a}{p}\right)_n^{i_f^o} \sum_{z=1}^{n}\left\{\frac{1 + r_e}{1 + i_f}\right\}^z \quad (30)$$

2. Repairs and Maintenance

Although this is an important item, little is known of the relation with the operating parameters such as speed and belt width. For overall estimates it is often taken as some percentage of the overall cost of capital items plus some percentage of the belt cost [10]. This gives no insight into the variation with operating parameters. Although intuitively on general grounds it may be expected that maintenance cost and costs associated with overall reliability would increase with the speed of the conveyor, such variation has not been included because of ignorance of the form of the relation.

3. Labour

In comparing the belt conveyor with an alternative for a particular application, labour costs for operation are quite significant. In optimizing a belt conveyor for a particular application, the labour is unlikely to change with a different choice of operating parameters. For this reason no labour costs have been included.

4. Capital Cost Items

For all these items it has been found possible, over restricted ranges, to express the first cost as a linear function of the operating and design variables [3, 4]. The annual equivalent capital cost is obtained by multiplying the first cost by a coefficient determined on the basis of the economic analysis previously described. The annual equivalent cost relationships* may be summarised as follows:

Motor

$$I_2 = k_2(c_1 + c_2 P_M) \quad (31)$$

Gear Reducer

$$I_3 = k_3(c_3 + c_4 T_R) \quad (32)$$

Conveyor Belting

$$I_4 = k_4(c_5 + c_6 B) KL \quad (33)$$

Idler Pulleys

$$I_5 = k_5(c_7 + c_8 B)\frac{L}{a_o} \quad (34)$$

for carrying side

$$I_6 = k_6(c_9 + c_{10} B)\frac{L}{a_u} \quad (35)$$

for return side

In the above equations $c_1, c_2 \ldots c_{10}$ are first cost coefficients; $k_2, k_3 \ldots k_6$ are annual equivalent cost coefficients; P_M is the power; T_R is the transmitted torque; B is the belt width; a_o and a_u are spacing of idlers on carrying and return side respectively; L is the conveyor length. The factor K in equation (33) allows for the total belt length, taking into account such factors as take-up pulleys and trippers; ($K \geq 2$). It should be noted that the coefficients c_5 and c_6 for the conveyor belting are a function of the belt type and number of plies.

Other capital cost items include the belt tensioning arrangements, discharge arrangements and drive couplings. Although the cost of these items is dependent, to some extent, on the conveyor width, capacity and operating parameters, as far as the overall cost is concerned their contribution may be assumed constant. For this reason they need not be included in the cost or objective function. As shown by Heaney [11], the conveyor structure, which is a function of belt width, is a major component of the overall cost and should be taken into consideration.

The overall cost or objective function is the summation of all the component costs, as indicated by equation (9).

5. Belt Conveyor Examples

5.1 Design of Single Conveyor

Consider the problem discussed in reference [3] where a belt conveyor, 500 m long, is required to convey a bulk material of bulk density $\varrho_m = 800$ kg/m³ up an incline of 1 in 10 and discharge it at a rate of $Q_m = 600$ t/h.

1. Principal Design Assumptions

- Idlers — 3 roll system with $\lambda = 1$ and $\beta = 35°$
- Surcharge angle $\delta = 20°$
- Belt type — Kuralon/Nylon (Type KN 150)
  Allowable stress $\sigma_b = 15.8$ kN/m, ply
- Gear reducer — helical type
- Operation — 12 hours/day over 300 days per year

2. Design Constraints

Belt width $0.65 \leq B \leq 2.0$ (m)
Plies $2 \leq p \leq 8$
Belt speed $0.5 \leq v \leq 6$ (m/s)

3. Economic Considerations

The basic assumptions are:

- General inflation rate is 10 %.
- Energy costs — unit cost of energy is $0.0314 per kW/h — annual cost escalation rate is 15 %.
- Installation — life of the conveyor and drive components is 12 years. Salvage value is zero. Cost escalation rate per year for drive and structure is 10 %.
- Conveyor belting and idler pulleys — life of 7 years is assumed with zero salvage value. The belt and idlers are replaced after the seventh year and are then depre-

* All costs are expressed in dollars.

ciated and written off at the end of the twelfth year. The price escalation rate per year for these components is 10%.

— After tax, rate of return on equity capital is 5%.

— Taxation rate is 0.46.

— Depreciation by straight line method.

With these assumptions the annual equivalent cost coefficients are:

— Energy $k_1 = 1.3165$
— Motor $k_2 = 0.1664$
— Gear reducer $k_3 = 0.1664$
— Conveyor belting $k_4 = 0.2593$
— Carrying idlers $k_5 = 0.2593$
— Return idlers $k_6 = 0.2593$
— Structure $k_7 = 0.1664$

The method of calculating the above coefficients is illustrated in the Appendix.

4. Design Solutions

To gain some appreciation for the cost variations involved by considering alternative design solutions, cost functions have been computed for a range of belt widths. The results are shown in Fig. 3. Fig. 3 (a) shows the variation of velocity, power and number of plies as a function of belt width; the corresponding annual equivalent cost curves are presented in Fig. 3 (b).

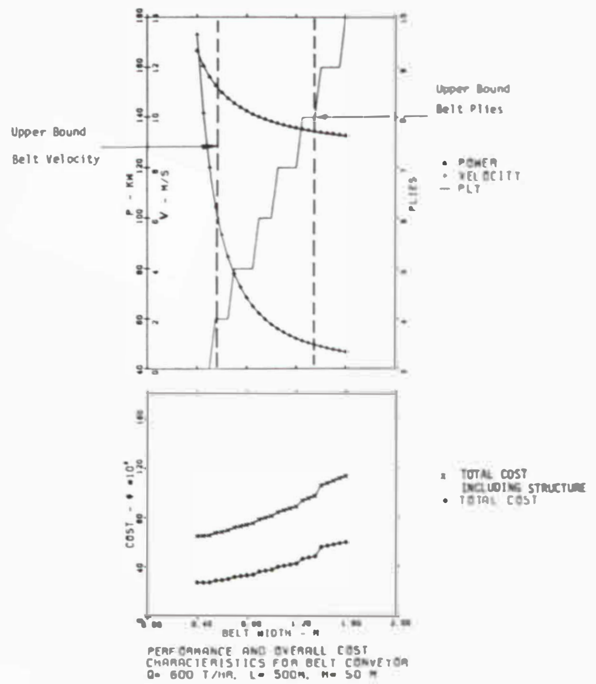

Fig. 3: Performance and overall cost characteristics for belt conveyor $Q = 600$ t/h, $L = 500$ m, $H = 50$ m

In the solution of the conveyer design problem consideration needs to be given to the constraints which have been previously indicated, as well as to the need for some of the design variables to be discrete values. A possible solution in this case is:

Belt width B = 0.65 m (a preferred size)
Number of plies = 4
Belt speed = 4.47 m/s
Power = 147 kW

Total annual operating cost

I_C = \$30,050 without structure
I_T = \$69,880 with structure included

It is useful to examine the contributions of each item in the overall cost. This information is presented in Fig. 4 and, as can be seen, the over-riding contribution is that due to the structure. With respect to the actual conveyor components,

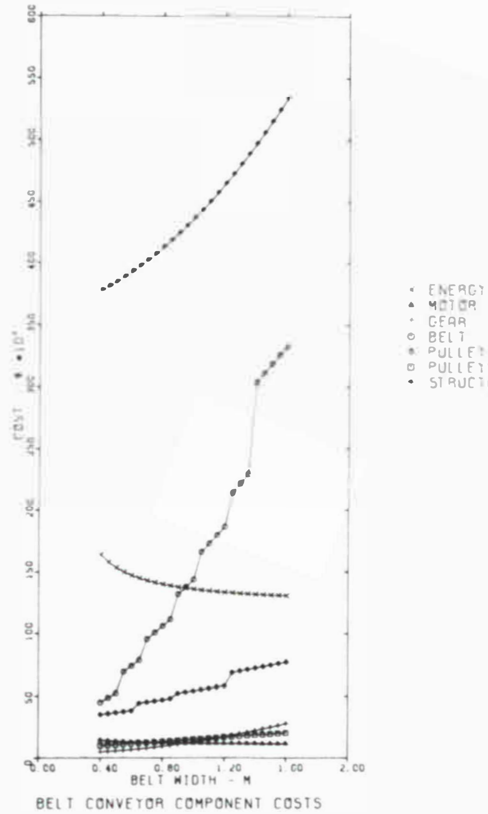

Fig. 4: Belt conveyor component costs

the substantial cost is that due to the belt. The example clearly demonstrates the advantages of using narrower, faster running belts. For instance, doubling the belt width reduces the required belt speed but results in a considerable cost increase.

To illustrate the effects of variations due to cost escalation, the same problem was examined for the case where the annual cost escalation rate is 20% for the energy costs and 15% for the belt, all other parameters being the same as before. In this case

$k_1 = 1.7510$

and

$k_4 = 0.2983$

(see Appendix for computation of this coefficient)

The comparable annual equivalent costs for the conveyor alone and for the conveyor plus structure are, respectively,

I_c = \$ 35,830
I_T = \$ 75,660

5.2 Multiple Conveyor Systems

Where materials are to be conveyed by belts over long distances, multiple conveyors, as illustrated in Fig. 1 (f), need to be employed. The choice of the number of conveyors and the individual length of each component conveyor will often be dictated by the design constraints such as the need to limit the maximum belt tension to suit the maximum number of plies available. However, it is also evident that in view of the cost variations per unit length, it is important that economic factors are taken into account.

By way of illustration, the annual equivalent costs per unit length of conveyor (neglecting the structure) have been determined for the case where the throughput is Q_m = 600 t/h, bulk density ρ_m = 800 kg/m³ and the slopes are zero in one case and 1 in 10 in another. All other relevant parameters are the same as in the previous example. Figs. 5 and 6 show, for the two cases, the annual equivalent cost per metre length as a function of belt length for a range of belt widths. The range of belt lengths considered is from zero to one kilometre.

In both cases the costs per unit length for very short length conveyors are very high, as would be expected. For the zero slope case (Fig. 4) and for the range of lengths beyond 200 m, the costs per unit length are substantially constant. This trend is also shown to occur for the slope of 1 in 10 (Fig. 5)

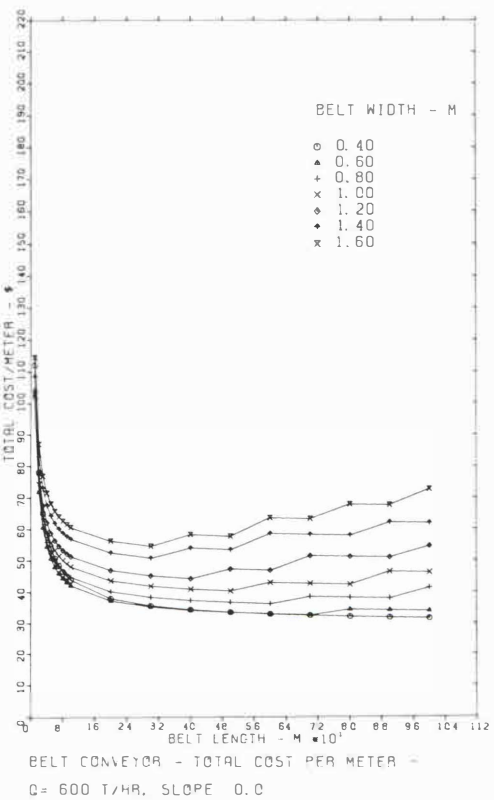

Fig. 5: Belt conveyor — total cost per metre — Q = 600 t/h, slope = 0.0

for the narrower belts, but once the belt widths increase beyond 0.8 m, the cost per unit length starts to increase appreciably with belt length. For wider belts the indications favour the use of several short conveyors rather than one long belt. However, the cost advantage in employing several conveyors would be either partially or totally offset by the additional costs due to the transfer stations.

6. Optimization of Screw Conveyors and Bucket Elevators

The procedures described in this paper may be readily applied to other types of conveyors. Some work relating to the optimum design of enclosed screw or auger conveyors and bucket elevators has already been performed by Roberts et al. [1, 2, 3]. A typical set of optimum performance curves for the screw conveyor is shown in Fig. 7 while the component cost contributions for a typical bucket elevator are presented in Fig. 8. These two sets of curves have been calculated using the same set of economic performance criteria; both sets of curves do not include the cost of conveyor structure.

The performance requirements are:

- Throughput Q_m = 51 t/h
- Unit Height of Lift = 5 m
- Material Conveyed = Wheat

For the screw conveyor the optimum solution is:

α = 80°
D = 0.207 m
L = 7.42 m
N = 503 rev/min
P_m = 6.17 kW
I = $ 503/annum

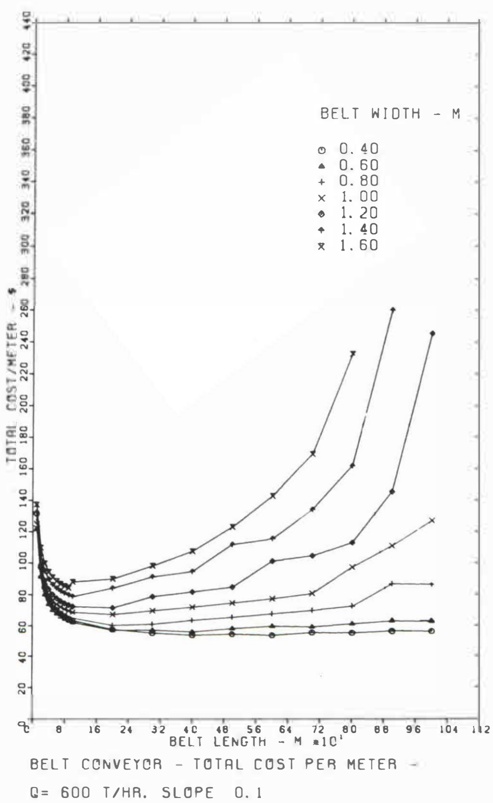

Fig. 6: Belt conveyor — total cost per metre — Q = 600 t/h, slope = 0.1

For the bucket elevator, choosing a belt speed of v = 2 m/s, the relevant details are:

B = 0.2 m (minimum)
v = 2 m/s
No. of plies p = 3
Power = 1.85 kW
I = $ 515/annum

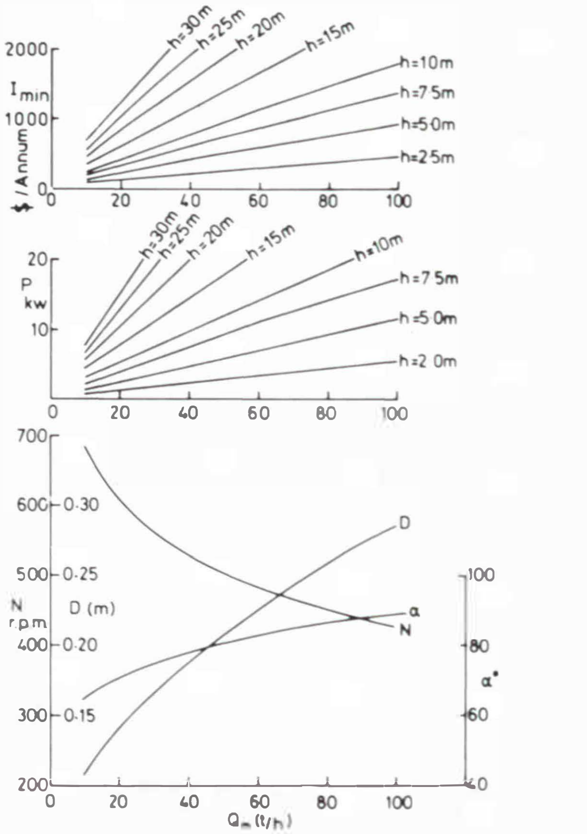

Fig. 7: Optimum performance characteristics for enclosed screw conveyor

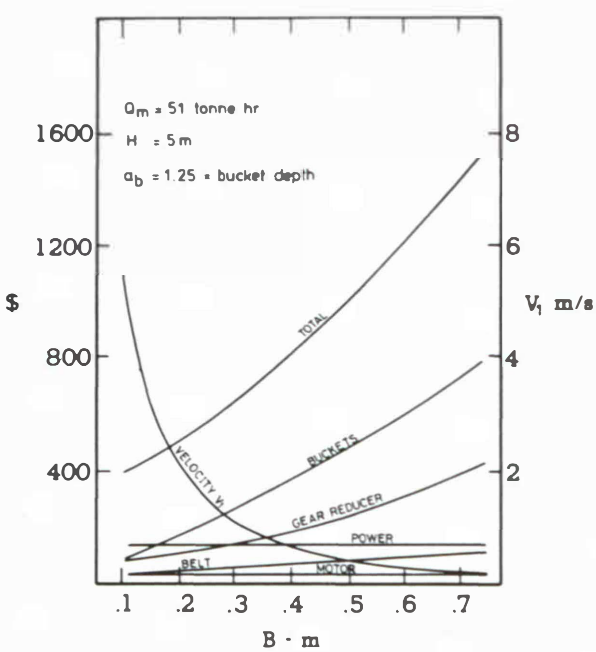

Fig. 8: Bucket elevator cost and performance characteristics

Thus it can be seen that the annual equivalent costs based on the conveyor components are quite similar. For the total cost, the annual equivalent cost of the conveyor structures needs to be added to the above amounts. The structural costs would be expected to be higher in the case of the bucket elevator in view of the casing and associated components. Nonetheless it would be expected that the total costs of the two conveyors would be of the same order.

7. Conclusions

This paper has drawn attention to the costs of bulk handling operations and the consequent need to design more efficient and economical systems. To achieve this objective a design methodology has been developed which integrates the underlying principles of engineering economic analysis with the concepts of optimization theory.

By way of example, the paper has dealt with the economic analysis and optimum design of belt conveyors for bulk solids handling. The various conveyor component costs as functions of the overall costs have been examined and the conclusions drawn favour the use of narrower, faster-running belts. Analysis of annual equivalent costs per unit length has provided guidelines for selection of optimum lengths of conveyors in multi-conveyor systems.

On the basis of the design and analysis procedure presented, comparisons between various types of conveyors can readily be made. It is clear, from the results presented, that the global problem of optimization applied to large, integrated handling systems can only be meaningful when the best operating conditions of individual conveyors and other items of handling equipment are understood.

References

[1] Roberts, A. W., and Charlton, W. H., "Optimum Design and Control of Grain Conveyor Systems", Proceedings of the International Grain and Forage Harvesting Conference, American Society of Agricultural Engineers (ASAE) and Commission Internationale du Genie Rural (CIGR) held at Iowa State University, USA, Sept. 1977

[2] Roberts, A. W., Scott, O. J., and Hayes, J. W., "Design Criteria for Optimum Performance of Conveyor Systems used in Agricultural Production", Proceedings of Conference on Agricultural Engineering, The Institution of Engineers, Toowoomba, Qld., Australia Aug. 1978

[3] Roberts, A. W., Hayes, J. W., and Scott, O. J., "Economic Considerations in the Optimum Design of Conveyors for Bulk Solids Handling", Proceedings of International Powder and Bulk Solids Handling and Processing Conference, Philadelphia, USA, May 15—17, 1979, pp. 101—116

[4] Roberts, A. W., and Hayes, J. W., "Economic Analysis in the Optimum Design of Conveyors", TUNRA, The University of Newcastle, N.S.W., Australia, 1980 (ISBN 0 7259 340 6)

[5] Kuester, J. L., and Mize, J. H., "Optimization Techniques with Fortran", McGraw Hill, 1973

[6] Khaw, B. W., "Evaluation of Computing Algorithms for Solving Non-Linear Multi-Variable Optimization Problems", Unpublished B. E. Thesis, Dept. of Mech. Eng., The University of Newcastle, N.S.W., Australia, 1977

[7] Beveridge, G. S. G., and Schechter, "Optimization Theory and Practice", McGraw Hill. 1970

[8] Lim, B. C., "Optimization of Bulk Solids Handling Systems", Unpublished B.E. thesis, Dept. of Mech. Eng., The University of Newcastle, N.S.W., Australia, 1979

[9] ISO/DIS 5048 Draft International Standard, Continuous Mechanical Handling Equipment — Belt Conveyors with Carrying Idlers — Calculation of Operating Power and Tensile Forces

[10] Cummins, A. B., and Given, I. A., "SME Mining Engineering Handbook", Vol. 2, The American Institute of Mining, Metallurgical and Petroleum Engineers Inc., 1973

[11] Heaney, S. J., "An Investigation into the Design of Belt Conveyors", Unpublished M.Eng.Sc. Thesis, Dept. of Mech. Eng., The University of Newcastle, N.S.W., Australia 1979

[12] Smith, G. W., "Engineering Economy: Analysis of Capital Expenditures", 2nd Ed., Iowa State University Press, 1973

Appendix:

Calculation of Annual Equivalent Cost Coefficients.
Example — Coefficient for Belt Cost, k_4.

Assumptions

Capital is all equity capital, that is $r_d = 0$
Required return on equity, $i_e = 5\%$
Income tax rate, $t = 46\%$
Conveyor system life, $n = 12$ years
Estimated belt life = 7 years
General inflation rate $r = 10\%$
Annual cost escalation rate of belt $r_b = 15\%$
Salvage value at any time, $V = 0$
Depreciation by straight line based on 7 year life with remaining value written off at the end of year 12

$$i_f = (1-t)\, r_d\, i_d + (1-r_d)\,[(1+r)(1+i_e) - 1]$$
$$= 0 + (1 + .10)(1 + .05) - 1$$
$$= 0.155$$

$$i_f^o = \frac{(1-t)\, r_d\, i_d - r r_d}{1 + r} + (1-r_d)\, i_e$$
$$= 0 + 1\,(.05)$$
$$= .05$$

Price of replacement belt $= A\,(1.15)^7$

Present equivalent of first cost of belts

$$= A\left[1 + \left(\frac{1.15}{1.155}\right)^7\right]$$
$$= 1.9701\,A$$

Depreciation of first belt $= \frac{A}{7}$ each year for seven years

Depreciation of second belt $= \frac{A}{7}(1.15)^7$ each year plus an additional $\frac{2A}{7}(1.15)^7$ in year 12.

$$PED = \frac{A}{7}\left(\frac{p}{a}\right)_7^{i_f} + \frac{A}{7}(1.15)^7 \left(\frac{f}{a}\right)_5^{i_f}\left(\frac{p}{f}\right)_{12}^{i_f} + \frac{2A}{7}(1.15)^7\left(\frac{p}{f}\right)_{12}^{i_f}$$

$$= A\left[\frac{(1.155)^7 - 1}{(7)(.155)(1.155)^7} + \frac{(1.15)^7}{7}\left(\frac{1.155^5 - 1}{.155}\right)\frac{1}{(1.155)^{12}} + \frac{2}{7}(1.15)^7\frac{1}{(1.155)^{12}}\right]$$

$$= A\,[.5855 + .4591 + .1348]$$
$$= 1.1794A$$

$$PEC = \frac{A - V\left(\frac{p}{f}\right)_n^{i_f} - t\,(PED)}{1 - t}$$

$$= \frac{1.9701A - .46\,(1.1794A)}{.54}$$
$$= 2.6436A$$

$$AEC = PEC\left(\frac{a}{p}\right)_{12}^{5}$$

$$= (2.6436)(.11283)A$$
$$= .2983A$$

Thus the annual equivalent cost coefficient

$$k = \frac{AEC}{A} = 0.2983$$