

Determination of the Mechanical Properties of Cohesive and Non-Cohesive Powdered Materials

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Die Bestimmung der mechanischen Eigenschaften kohärenter und nicht-kohärenter pulverförmiger Massen
Détermination des caractéristiques mécaniques intéressantes les milieux pulvérulents avec ou sans cohésion
Determinación de las características mecánicas interesando los medios pulverulentos con o sin cohesión

粉末材料特性の確定

粉末物质特性的测定

تحديد خصائص المواد المسحوقة

Die Bestimmung der mechanischen Eigenschaften kohärenter und nicht-kohärenter pulverförmiger Massen

Die Kenntnis der mechanischen Eigenschaften von in Silos gelagerten pulverförmigen Massen ist erforderlich für die Bestimmung der Silowanddrücke und der Drücke auf sonstige Konstruktionselemente von Schüttgut-Förderanlagen.

Die experimentelle Bestimmung des Böschungswinkels pulverförmiger Medien ist bekannt, ebenso die Bestimmung der Hauptspannungen mit Hilfe des Mohr'schen Kreises unter Zuhilfenahme triaxialer Spannungsuntersuchungen.

Bisher ist es jedoch nicht möglich gewesen, den Zusammenhang zwischen natürlichem Böschungswinkel und dem Winkel der inneren Reibung mathematisch zu bestimmen. Dies ist das Thema des vorliegenden Berichtes. Die vorgelegten Ergebnisse erlauben die

- direkte Bestimmung des Winkels der inneren Reibung und des kleinsten Reibungswinkels der inneren Reibung ohne Benutzung des Mohr'schen Kreises sowie
- die Kontrolle von Meßergebnissen und die Korrektur deren Abweichungen.

Determinación de las características mecánicas interesando los medios pulverulentos con o sin cohesión

El conocimiento de las características mecánicas de las materias ensiladas o manipuladas es fundamental para el cálculo de los esfuerzos que éstas ejercen sobre las paredes de los silos o sobre los órganos de ciertas obras y aparatos de manutención.

Ahora bien, sabemos determinar, experimentalmente, el valor del ángulo de talud natural de las materias pulverulentas, y merced al aparato triaxial, los valores de las tensiones principales que permiten determinar, mediante un trazado clásico de círculos de Mohr, el valor del ángulo de rozamiento interno.

Pero, hasta la fecha, no habíamos podido establecer la relación que existe entre el ángulo de talud natural y el ángulo de rozamiento interno. Esta relación, que es objeto del presente artículo, permite de aquí en adelante:

- calcular directamente los valores del ángulo de rozamiento interno y del ángulo de rozamiento interno mínimo sin recurrir al trazado de círculos de Mohr.
- controlar los resultados de las medidas realizadas, corrigiendo sus dispersiones eventuales.

Summary

Knowledge of the mechanical characteristics of materials contained in silos is fundamental for computation of the stresses exerted on silo walls or on construction members of various material handling installations.

We know how to determine experimentally the value of the angle of natural slope of pulverulent materials, and thanks to triaxial testing equipment, also the value of the principal stresses which allow the determination of the angle of internal friction by using the traditional Mohr circle method.

But it has never been possible until now to establish the relationship between the angle of natural slope and the angle of internal friction. This relationship is the subject of this report. It allows

- the direct computation of the angle of internal friction and the angle of minimum internal friction without applying the Mohr circle method and
- the control of measurement results and the correction of their possible spread.

Détermination des caractéristiques mécaniques intéressantes les milieux pulvérents avec ou sans cohésion

La connaissance des caractéristiques mécaniques des matières ensilées ou manutentionnées est fondamentale pour le calcul des efforts que celles-ci exercent sur les parois des silos ou sur les organes de certains ouvrages et appareils de manutention.

Or, on sait déterminer, expérimentalement, la valeur de l'angle de talus naturel des matières pulvérentes, et, grâce à l'appareil triaxial, les valeurs des contraintes principales qui permettent de déterminer, par un tracé classique de cercles de Mohr, la valeur de l'angle de frottement interne.

Mais on n'avait pu établir, jusqu'ici, la relation existant entre l'angle de talus naturel et l'angle de frottement interne. Cette relation, qui fait l'objet du présent article, permet désormais:

- de calculer directement les valeurs de l'angle de frottement interne et de l'angle de frottement interne minimum sans avoir recours au tracé de cercles de Mohr,
- de contrôler les résultats des mesures effectuées, et de corriger leurs dispersions éventuelles.

Nomenclature

γ	Density (kg/m^3)
β	Angle of natural slope
ϕ_0	Minimum angle of internal friction
ϕ	Angle of internal friction
ϕ'	Friction angle relating to lined silos and hoppers
C_a	Apparent cohesion factor
C_{\min}	Minimum cohesion factor
K_A	Maximum thrust coefficient

1. Introduction

At the International Conference on the Design of Silos for Strength and Flow held at the University of Lancaster, England, in September 1980, these authors emphasised the relevance and importance of accurate knowledge of the structural anisotropy of powdered solid materials, whether cohesive or non-cohesive and the accurate determination of various physical and mechanical parameters.

At another conference [6] these authors also indicated that there are several important ratios and functional relationships between the parameters β , ϕ , ϕ_0 , and the principal stresses σ_1 and σ_3 appertaining to the properties of powdered materials which can be determined using triaxial tests.

Concerning the study of structural anisotropy of a pulverulent medium, these authors were able to demonstrate clearly the functional relationship between the apparent cohesion and the minimum cohesion [7].

The object of this paper is to summarize the main points and conclusions of these previous communications [1, 4, 5], in order to draw up practical design guidelines simplifying considerably the task of the design engineers, particularly those involved with soil mechanics and silo design and who typically utilise the triaxial compression test method for the determination of the parameters: ϕ , ϕ_0 , C_a , C_{\min} , according to values of principal stress σ_1 and σ_3 determined either in laboratory experiments on prepared samples or in situ.

Material density is utilised in the definition of the mechanical characteristics of the soil because its determination does not present any special difficulties.

2. Angle of Natural Slope

Terzaghi and Peck [2] define the angle of natural slope β as corresponding to the limit balance of materials without stress, its value being equal to the minimum internal friction angle ϕ_0 .

Harr [3], relating the application of probabilism to soil mechanics, contributes support thereto whilst setting forth [3, page 259] an ultimate friction angle ϕ_{cv} which he indicates and defines as being equal to β . This so called ultimate angle corresponds to the minimum angle of internal friction ϕ_0 .

Hence:

$$\beta = \phi_0 \quad (1)$$

3. Minimum Angle of Internal Friction ϕ_0

Harr [3] states that in the traditional outline of the Mohr circle for stresses relating to a powdered mass with a free horizontal surface retained by a vertical screen, principal stresses σ_1 and σ_3 [respectively σ_z and $K_A \sigma_z$ in [3] Fig. 8—14 page 271], defining the diameter of the Mohr circle, are such that the value of their ratio σ_3/σ_1 [or $K_A \sigma_z/\sigma_z$] corresponds to the value of the maximum thrust coefficient K_A .

The authors have determined via extensive experimentation and mathematical manipulation that the value of the maximum thrust coefficient is given by [4]:

$$K_A = \left[\frac{\pi - 2\phi_0}{\pi + 2\phi_0} \right]^2 \quad (2)$$

ϕ_0 , the *minimum angle of internal friction*, as defined above, is equal to the natural slope angle β .

With reference to the work by Harr [3]:

$$\frac{\sigma_3}{\sigma_1} = \left[\frac{\pi - 2\phi_0}{\pi + 2\phi_0} \right]^2 \quad (3)$$

Hence the value of ϕ_0 follows:

$$\phi_0 = \frac{\pi}{2} \cdot \frac{1 - \sqrt{\frac{\sigma_3}{\sigma_1}}}{1 + \sqrt{\frac{\sigma_3}{\sigma_1}}} \quad (4)$$

This expression allows the direct computation of the *angle of minimum friction* ϕ_0 once the value of the stresses σ_1 and σ_3 have been determined by triaxial tests.

An example of a practical application is given in the Appendix utilising the results of a test undertaken by the geotechnical laboratory of Bureau VERITAS.

4. Angle of Internal Friction

Direct computation of the angle of internal friction ϕ of a powdered solid material is feasible after the values of principal stresses σ_1 and σ_3 relating to the materials have been determined by triaxial compression tests [3, page 267].

5. Application to Cohesive and Non-Cohesive Materials

5.1 Non-Cohesive Powdered Materials

The principal stresses σ_1 and σ_3 having been determined, the angle of the tangent to the Mohr circle [which has a diameter equal to $\sigma_1 - \sigma_3$, the centre of which is at a distance from the origin 0 equal to $(\sigma_1 + \sigma_3)/2$], (Fig. 1), with the axis of the abscissa, represents the internal friction angle parameter ϕ .

The angle of internal friction is then derived from:

$$\sin \phi = \frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3} \quad (5)$$

5.2 Cohesive Powdered Solids

After the principal stresses σ_1 and σ_3 have been determined by an initial test and then subsequently by a second test, the angle of the tangent to the Mohr circles [having respectively diameters $\sigma_1 - \sigma_3$ and $\sigma_1' - \sigma_3'$, the centres of which are at respective distances from the origin equal to $(\sigma_1 + \sigma_3)/2$ and $(\sigma_1 + \sigma_3)/2$] and the axis of the abscissa represents the angle of internal friction ϕ (Fig. 2).

Hence from Fig. 2

$$\sin \phi = \frac{(\sigma_1' - \sigma_3') - (\sigma_1 - \sigma_3)}{(\sigma_1' + \sigma_3') - (\sigma_1 + \sigma_3)} \quad (6)$$

the angle of internal friction can be derived.

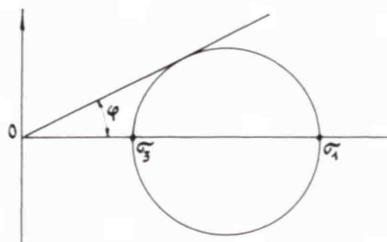


Fig. 1

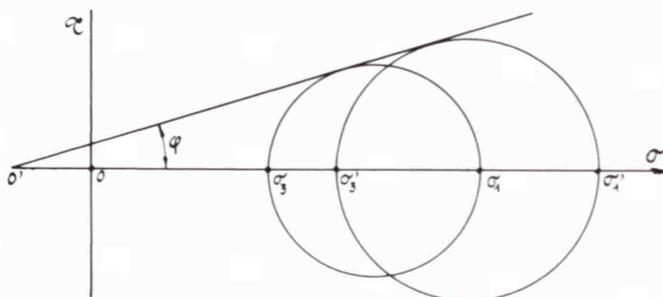


Fig. 2

6. Relationship between the Angle of Minimum Internal Friction ϕ_0 and the Angle of Internal Friction ϕ

The study of structural anisotropy of powdered solids demonstrates that stresses relating to an isotropic material, to which the theory of the Mohr circle applies, should be reduced in a given ratio to define an ellipse, here defined as an anisotropic ellipse, of stresses relating to the anisotropic powdered material.

The anisotropic ellipse is defined mathematically as follows: (with origin at centre)

$$\frac{x^2}{\left[\frac{\sigma_1 - \sigma_3}{2} \right]^2} - \frac{y^2}{\sigma_1 \sigma_3 \tan^2 \phi_0} - 1 = 0 \quad (7)$$

Hence the corresponding anisotropic coefficient is given by:

$$\lambda = \cos \theta = \frac{2 \tan \phi_0 \sqrt{\sigma_1 \cdot \sigma_3}}{\sigma_1 - \sigma_3} \quad (8)$$

θ being the slope angle of the plane which would contain the Mohr circle compared with the plane on which the latter would be projected to define the anisotropic ellipse.

The expression of the anisotropic coefficient is equivalent to $\tan \phi_0 / \tan \phi$ and this equivalence allows the approximate determination of the following relationship between the minimum angle of internal friction ϕ_0 and the angle of internal friction ϕ of the powdered material.

$$\phi_0 \cong 0.8 \phi \quad (9)$$

or

$$\phi \cong 1.25 \phi_0 \quad (10)$$

7. Angle of Friction ϕ' of a Powdered Material Against a Hopper/Silo Lining

The definition of the friction angle ϕ' of a powdered material in contact with a silo/hopper lining is accounted for by:

- Computing the static pressure on the silo lining,
- Computing the stability of the retaining wall. However, it is not necessary to calculate the value of the force exerted by the mass on the wall as the latter is not influenced by the state of the surface of the silo/hopper wall lining [1, 4, 5].

Determination of the friction angle ϕ' is undertaken in the laboratory and is not related in any way to the interpretation of stress measurements using triaxial compression tests.

8. Minimum Cohesion of Cohesive Powdered Materials

The representation of stresses relating to an isotropic powdered material with the help of the Mohr circle allows the determination of both the value of the apparent cohesion C_a of the material and that of the internal friction angle ϕ .

The elliptic representation of stresses relating to the anisotropic powdered material obtained by the application of the inherent stresses to the anisotropic coefficient (Eqn. 8) allows the determination of both the value of the minimum internal friction angle ϕ_0 and that of the minimum cohesion C_{min} (Fig. 3).

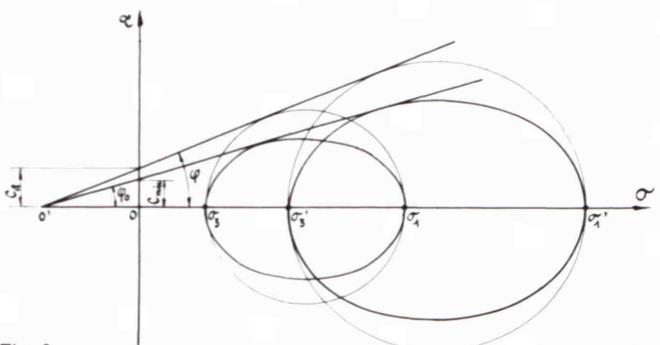


Fig. 3

The minimum cohesion value is the only parameter necessary in the computation of stability providing operational conditions remain constant.

Hence from Fig. 3 it can be seen that

- the value of *apparent cohesion* relating to the isotropic material is given by:

$$C_a = \left[\frac{\sigma_1 - \sigma_3}{2 \sin \phi} - \frac{\sigma_1 + \sigma_3}{2} \right] \tan \phi \quad (11)$$

- the value of the *minimum cohesion* relating to an anisotropic powdered material is given by:

$$C_{\min} = \left[\frac{\sigma_1 - \sigma_3}{2 \sin \phi} - \frac{\sigma_1 + \sigma_3}{2} \right] \tan \phi_o \quad (12)$$

9. Conclusions

Having determined, on the one hand, the value of the natural slope angle β and, on the other hand, the principal stresses σ_1 and σ_3 with the triaxial compression apparatus, we can derive the following parameters mathematically:

- the angle of internal friction ϕ
- the minimum internal friction angle ϕ_o
- the minimum cohesion C_{\min}

without direct use of Mohr circles or anisotropic ellipses but only by applying Eqns. (1) and (4) in case of a non-cohesive powdered material and Eqns. (6), (10) and (12) in the case of a cohesive powdered material.

10. Final Comment

From the above it may be concluded that, in the case of a cohesive powdered material, it is possible to deduce the following parameters: (Tables 1 and 2)

- the angle of *internal friction minimum*:

$$\phi_o = \beta$$

- the angle of *internal friction*:

$$\phi \approx 1.25 \beta$$

providing the natural angle of slope is known.

Table 1:
Summary of relationships and required formulae

Computed/ derived parameters	Parameters required	Required formulae
Non-Cohesive Materials		
ϕ_o	β	(1)
ϕ_o	σ_1 and σ_3	(4)
ϕ	σ_1 and σ_3	(5)
ϕ	ϕ_o	(9)
ϕ_o	ϕ	(10)
Cohesive Materials		
ϕ	σ'_1 and σ'_3	(6)
ϕ	ϕ_o	(9)
ϕ_o	ϕ	(10)
C_{\min}	ϕ_o and σ_1, σ_3	(12)

Table 2:
Summary of formulae

$$(1) \quad \beta = \phi_o$$

$$(4) \quad \phi_o = \frac{\pi}{2} \cdot \frac{1 - \sqrt{\frac{\sigma_3}{\sigma_1}}}{1 + \sqrt{\frac{\sigma_3}{\sigma_1}}}.$$

$$(5) \quad \sin \phi = \frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3}$$

$$(6) \quad \sin \phi = \frac{(\sigma'_1 - \sigma'_3) - (\sigma_1 - \sigma_3)}{(\sigma'_1 + \sigma'_3) - (\sigma_1 + \sigma_3)}$$

$$(9) \quad \phi_o \approx 0.8 \phi$$

$$(10) \quad \phi \approx 1.25 \phi_o$$

$$(12) \quad C_{\min} = \left[\frac{\sigma_1 - \sigma_3}{2 \sin \phi} - \frac{\sigma_1 + \sigma_3}{2} \right] \tan \phi_o$$

Appendix

Results of tests undertaken at the geotechnical laboratory of Bureau VERITAS

Date of test: December 4, 1973

Material: Fine siliceous sand

Report No. 80919

1. Angle of natural slope (average of measurements):
 $\beta = 33.50^\circ$

2. Internal friction angle: $\phi = 41.78^\circ$
(dimensions of tested samples: diameter = 70 mm,
height = 200 mm)

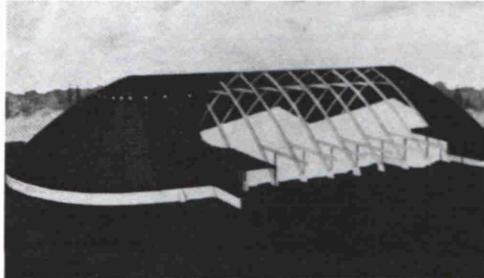
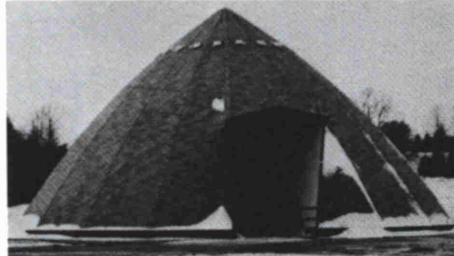
Overall hydrostatic pressure (bars)	Maximum axial stress (bars)	$\sin \phi$ tangent to circles	ϕ
σ_3	σ_1		
1.00	5.025		
2.00	10.00	0.666	41.78°

Hence the test actually confirms the value of the angle of natural slope β compared with the value of the internal friction ϕ : the ratio β/ϕ is practically equal to 0.8, thus verifying Eqns. (9) and (10):

$$\beta = 0.801 \phi \quad \text{or} \quad \phi = 1.247 \phi_0 \cong 1.25 \phi_0$$

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