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Hopper Studies at Cambridge University, England: A Review of Ten Years Progress

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دراسات القواديس بجامعة كامبريدج . انجلترا : استعراض الانجازات التي تمت في هذا المجال خلال عشر سنوات

10 Jahre Forschung über Auslauftrichter an der Cambridge University, England

Dieser Beitrag gibt einen ausführlichen Überblick über die in der Zeit von 1970 bis 1980 durchgeführten Forschungsvorhaben auf dem Gebiet der Auslauftrichter am Department of Chemical Engineering der Cambridge University. Die Forschungsgruppe hat sich auf Aspekte der Spannungsverteilung in Trichtern, auf die Abzugsraten durch Auslauf-Öffnungen, auf die Geschwindigkeitsverteilung sowie auf die Auswirkungen zwischenräumlicher Druckgefälle spezialisiert und bedeutende Beiträge geleistet.

Etudes de trémies à l'Université de Cambridge, Grande-Bretagne: Examen des progrès réalisés en dix ans

Cet exposé présente un examen détaillé des progrès réalisés en dix années de recherche effectuées par le Département de Génie Chimique de l'Université de Cambridge concernant quatre aspects principaux de la conception de trémies. Le groupe a concentré son travail sur les secteurs suivants: distributions des tensions dans les trémies, vitesse de vidange à travers les orifices, distribution de la vitesse et effets des différences de pression interstitielles. Le groupe de recherches a apporté dans ces domaines une contribution importante.

Estudios de tolvas en la Universidad de Cambridge, Inglaterra: Examen de diez años de progreso

Este artículo presenta un resumen detallado de una década de progreso en la investigación realizada en el Departamento de Ingeniería Química de la Universidad de Cambridge en torno a cuatro aspectos principales del diseño de tolvas. El grupo ha concentrado su labor en y a hecho importantes aportaciones a los conocimientos en las áreas de las distribuciones de tensión en las tolvas, las velocidades de descarga por orificios, la distribución de velocidades y los efectos de gradientes de presiones intersticiales.

Summary

This paper presents a detailed review of a decade of original research undertaken within the Department of Chemical Engineering at the University of Cambridge concentrating primarily on four main aspects of hopper design. The group has made substantial contributions to knowledge in the areas of stress distributions in hoppers, discharge rates through orifices, velocity distribution and in the effects of interstitial pressure gradients.

Notation

b	breadth of a slot orifice
B	kinematic constant
C	constant in Beverloo's correlation
d	particle diameter
D_o	orifice diameter
g	acceleration due to gravity
k	constant in Beverloo's correlation
K	$(1 + \sin\phi)/(1 - \sin\phi)$
l	length of a slot orifice
p	pressure
$\left.\frac{dp}{dr}\right)_o$	pressure gradient at the orifice
Q	volumetric flow rate per unit width
r_o	radius, from virtual apex to orifice
Re	Reynolds number
u	horizontal component of velocity
v	vertical component of velocity
W	mass flow rate
x	horizontal co-ordinate
y	vertical co-ordinate
z	axial co-ordinate
α	hopper half angle
ΔP	overall pressure difference
ζ	elliptic co-ordinate
ϱ	density
ϕ	angle of repose

1. Introduction

The behaviour of bulk solids in hoppers has been an active topic of research in the Department of Chemical Engineering at the University of Cambridge for the last ten years. During this period there have been six research students working in this field within the department and their work is discussed below. In addition to this, the regulations for the Chemical

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Engineering Tripos require that final year students undertake research projects. Such projects are expected to take about a fifth of the student's time for two terms and are normally undertaken by a pair of students. Projects are of necessity modest in scope and rarely produce results which are worth publishing in their own right. However, it is found that these projects are invaluable for testing out new ideas and for supplementing the work of research students. One of the objectives of this paper is to give such projects the publicity they deserve. The date of submission of the thesis or project report is given after the student's name. Where the results have been published references are given in square brackets.

The work of the group has been concentrated on the following four topics which are discussed separately below:

1. Stress distributions in hoppers.
2. Discharge rates through orifices.
3. Effects of interstitial pressure gradients.
4. Velocity distribution.

2. Stress Distributions

The classic stress analysis of Janssen [1] and later modifications, for example by Walker [2], Walters [3] and Enstad [4] involve unconfirmed assumptions about the stress distribution across some control surface and at the best can only be regarded as useful approximations. However for a Coulomb material the method of characteristics, as expounded by Sokolovskii [5] may be considered to be exact provided the material is in a state of incipient yield at all points as will be the case for slow discharge through a converging hopper. However, calculations using Sokolovskii's method are too lengthy for every-day use and one normally falls back on a modified form of Janssen's method.

Hancock [6] (Ph.D. 1970) developed Sokolovskii's method for the case of a parallel sided plane strain bunker and compared his results with those of approximate analyses. He considered the case when the top surface of the material was stress-free and concluded that the approximate methods gave results within a few percent of the exact calculations for a wide range of friction angles and therefore could be used with confidence despite their lack of rigour. Hancock also identified an error in Airy's [7] analysis for the plane strain bunker. When corrected this method also provided reasonable predictions.

Though the confirmation that the usual analyses are correct is valuable, Hancock's major contribution to the work of the group was the development of the computer programs necessary for the use of the method of characteristics. In particular he solved the complicated problems associated with the intersection of a stress-free top surface with a frictional wall. At the point of intersection there is a discontinuous change in the direction of the major principal stress which causes considerable arithmetic difficulties.

Horne [7, 8, 9], (Ph.D. 1976) built upon the foundation laid by Hancock and used the method of characteristics to solve a greater range of problems. He agreed with Hancock that the modified Janssen methods were good enough for a stress free top surface but found considerable errors in the presence of a surcharge. He also investigated the stress discontinuities that arise in solutions for the passive state and developed algorithms for the very formidable calculations involved. From these he was able to predict the stress distribution in converging hoppers and agreed with Jenike [10] that the stress distribution tends to that predicted by the radial stress field. However Horne disagreed with Jenike both with regard to the speed and form of the convergence to that stress field. In many cases he found that the exact solution approached that of the radial stress field very slowly thereby casting doubts on the utility of that convenient but approximate solution.

Horne was also able to investigate the switch stress caused at the junction of a converging base and a parallel sided bunker. His predictions suggest that Walters' analysis [3] of this problem is over-simplified. The switch surface was found to be highly curved, not planar as assumed by Walters, and that the maximum wall stress occurred somewhat below the switch surface. These techniques are currently being used to predict the stress distributions resulting from the insertion of obstacles into the granular medium.

Besides working on problems in plane strain Horne also considered axisymmetrical situations. However he was unable to obtain any reasonable predictions in this case. Even in the active state where the characteristics leaving the wall diverge initially, it was found that they converged as they approached the axis of symmetry. As a result stress discontinuities are formed internally but it was not found possible to predict precisely where they would occur. Furthermore a second family of discontinuities was found due to discontinuous changes in the circumferential normal stress. Horne concluded that the axisymmetric case was intractable by the method of characteristics.

Hancock also measured the wall stresses and found that for the static material the stresses were sensitive to the method of filling. For a discharging material for which the effects of elastic deformation are less important, greater agreement with theoretical predictions was obtained. However, the wall stresses were found to fluctuate rapidly about the predicted level with an amplitude comparable with the mean value. The cause of these fluctuations was not investigated but they are believed by Bransby [11] to be due to the periodic formation of rupture surfaces at the top of the converging section.

Recently Haslett and Johnston (Tripos 1980) have measured the forces on obstacles suspended in a discharging hopper. They found that the force increased with greater immersion in a manner similar to that predicted by the Janssen [1] analysis.

3. Discharge Rates through Orifices

None of the Ph.D. topics and only a few of the Tripos projects have been exclusively concerned with the measurement of discharge rates through orifices in the absence of interstitial pressure effects. However, in many of the projects concerned with velocity distribution the flow rate was determined by some orifice and it has always been found that the *Beverloo* [12] *correlation,*

$$W = C \varrho \sqrt{g}(D_o - kd)^{5/2} \tag{1}$$

where $C \cong 0.58$ when SI units are used, is the most reliable for coarse free-flowing materials.

In using the *Beverloo correlation* some doubt exists about the appropriate value of the density ϱ. The early project of Huntington and Rooney (Tripos 1970) investigated the effect of varying the initial packing of the material. They found no effect of the initial void fraction on the mass flow rate but did find that for a highly compacted bed there was a protracted period between the initiation of flow and the start of the descent of the top surface. This is due to the dilation of the material to some voidage characteristic of the flowing state. Huntington and Rooney definded a flowing density as the ratio of the mass flow rate to the volumetric flow rate calculated from the observed rate of descent of the top surface. This flowing density appeared to be a function solely of the nature of the material and to be independent either of the flow rate or the initial packing. The use of this density in the *Beverloo correlation* gives the least scatter in the constant C.

Myers and Sellers (Tripos 1977) and Laird and Roberts (Tripos 1979) investigated the effect of wall inclination on the discharge rate, a topic that seems to have received little attention since the original work of Rose and Tanaka [13]. These latter authors found that the flow rate was proportional to $(\tan\alpha \tan\phi)^{-0.35}$ provided $(\tan\alpha \tan\phi) < 1$. Here α is the inclination of the hopper wall to the vertical and ϕ is the angle between the stagnant zone boundary and the horizontal.

Working with a plane strain hopper and using only kale seed, Myers and Sellers found a similar result but with an index of —0.21 instead of the value of —0.35 as found by Rose and Tanaka. Laird and Roberts, using a conical hopper also found the same type of behaviour but could find no single index to correlate all their results. Values between —0.31 and —0.45 were required, the value seeming to be a function of the nature of the material. This topic clearly requires further study.

Myers and Sellers also noticed that the constant k in the Beverloo expression, representing the width of the *empty annulus* was a strong function of wall slope decreasing from the generally accepted value of about 2.0 for a flat bottomed bunker to around 0.7 when the walls were almost vertical.

They also found agreement with Glastonbury's [14] suggestion that for non-circular orifices the flow rate was proportional to $A\sqrt{D_h}$ where A is the area and D_h the hydraulic mean diameter. Modifying this to allow for the *empty annulus* gives for a $b \cdot l$ slot orifice ($b \ll l$)

$$W = \frac{4\sqrt{2}\,C}{\pi}\varrho\, g^{1/2} (l - kd)(b - kd)^{3/2} \tag{2}$$

Using Beverloo's value of C the predicted constant

$$\frac{4\sqrt{2}\,C}{\pi} = 1.04$$

was found to agree well with the experimental value of 1.03.

Theoretical work on this topic, carried out in conjunction with Prof. Davidson, Head of the department's fluidisation group, resulted in the so-called *Hour Glass Theory* [15]. Most previous theories have assumed, explicitly or implicitly, that the flow rate is independent of the quantity of the material in the hopper. In this work the effect of the top surface was not neglected but was shown to be unimportant except for very small angles of friction. The flow rates predicted by this theory are somewhat in excess of those found experimentally presumably because the effect of wall friction was neglected. Furthermore both for the conical and plane strain case the mass flow rate was predicted to be proportional to $\sin^{-1/2}\alpha$, a result not in perfect accord with the experiments described above.

The *Hour Glass Theory* also gives a prediction of the stress distribution. This is shown to be independent of the flow rate except in the immediate vicinity of the orifice; the departure from the static distribution being of order r^{-4}. This explains the well known experimental result that only the conditions immediately above the orifice affect the discharge rate.

An attempt by Myers and Sellers to allow for wall friction by a modification of Walker's analysis [2] met with little success.

4. Interstitial Pressure Effects

A more extensive series of projects has been concerned with the effect of interstitial pressure on the flow rate from hoppers. Most previous work on this subject has been concerned with correlating the mass flow rate with the pressure difference between the orifice and some arbitary point on the hopper wall. Since the theoretical prediction of mass flow rate suggests that only effects in the vicinity of the orifice are important, our work has been based on the idea that there should be a relationship between the mass flow rate and the pressure gradient at the orifice. The simplest approach is to assume that the pressure gradient produces a body force that can be added directly to the weight of the material. Following this idea the *Beverloo correlation* becomes

$$W = C\varrho \left\{ g + \frac{1}{\varrho} \left.\frac{dp}{dr}\right)_o \right\}^{1/2} (D_o - kd)^{5/2} \tag{3}$$

Harrison and Mushin (Tripos 1979) however modified the *Hour Glass Theory* and predicted that there should be a multiplicative factor of

$$\frac{2K - 3}{2K - 1}$$

before the term

$$\frac{1}{\varrho} \left.\frac{dp}{dr}\right)_o .$$

Since K is normally large this correction factor has little effect.

Much work has been done on the simplest geometry namely the conical hopper. At low percolation rates the Carman-Kozeny equation predicts that

$$\left.\frac{dp}{dr}\right)_o = \frac{\Delta P}{r_o}$$

where ΔP is the pressure difference between the orifice and a point at great distance from the orifice. However at higher gas percolation rates the inertial terms cannot be neglected and the *Ergun equation* must be used instead of the *Carman-Kozeny equation.* This predicts that

$$\left.\frac{dp}{dr}\right)_o = \frac{\Delta P}{r_o} f(\mathrm{Re})$$

where $f(\mathrm{Re}) \to 1$ at low gas *Reynolds* Numbers and $\to 3$ as $\mathrm{Re} \to \infty$. Thus is these basic ideas are correct there should be a linear relationship between W^2 and

$$\left.\frac{dp}{dr}\right)_o$$

but no simple relationship between W and ΔP. By measuring the pressure profile within the hopper by a series of wall tappings Harrison and Mushin (Tripos 1979) and later Bunt and Smith (Tripos 1980) found that plots of W^2 against

$$\left.\frac{dp}{dr}\right)_0$$

were linear whereas those of W^2 against ΔP were not. Bunt and Smith also found that the pressure profile could be well correlated by an equation of the *Ergun type.*

For the case of a cylindrical bunker with a central circular orifice, the relationship between overall pressure difference and pressure gradient at the orifice is much more complicated. At sufficiently low *Reynolds* numbers the pressure profile should satisfy *Laplace's equation* and hence recourse may be made to potential flow analyis. Near the orifice the result given by Lamb, [16], $p \propto \cot \zeta$, where ζ is the elliptic coordinate, may be used but far from the orifice the pressure will vary linearly with axial distance z. Bunt and Smith (Tripos 1980) studied the matching of these profiles using a numerical solution to *Laplace's* equation. It is hoped to continue this work and to extend it to cases where the inertial terms cannot be neglected.

The work described above has been concerned with cases in which air is supplied above the material. Altiner (Ph.D. 1975, supervised by Prof. Davidson) considered the case in which air was supplied through a porous section of the wall near the orifice. Enhanced flow rates were found as expected but there was an upper limit to the quantity of air that could usefully be injected. At greater flow rates the bed above the injection point became fluidised and the excess air simply escaped upwards without aiding the flow.

The combination of a hopper and a standpipe was studied by Coulson and Hamilton (Tripos 1977). The solids falling through the standpipe drag the air with them and thus create a low pressure at the orifice. A good correlation was obtained between the mass flow rate and the resulting pressure profile in the hopper but they did not succeed in predicting the pressure deficit from the behaviour of the accelerating particles in the standpipe.

In the cases described above the interstitial pressure profile resulted from the supply of air or the installation of a standpipe. If, however, the voidage of the material changes as it flows through the hopper, relative motion of air and particles must occur and this will result in a pressure gradient. From the *Carman Kozeny* equation this gradient will be inversely proportional to the square of the particle diameter and so will be of the greatest importance for fine powders. Crewdson and Ormond (Tripos 1975), [17] by making sweeping assumptions about the relationship between voidage and interparticle stress were able to obtain an analytic solution for the pressure distribution and hence the reduction in flow rate due to compressibility effects. They showed that these effects are negligible for particles of diameter greater than 500 μm but that for finer particles the mass flow rate is much less than that predicted by the *Beverloo expression.* Spink [18] (Ph.D. 1976), considered the same problem in greater detail and measured pressure and voidage profiles simultaneously. The latter proved far from straightforward and he was only able to show that the measured voidages were compatible with the observed pressure profiles. Whilst the work of Spink and of Crewdson and Ormond shows that compressibility effects are important in the determination of mass flow rates they concluded that little further progress in this field is likely until the relationship between void fraction and interparticle stress has been resolved satisfactorily.

5. Velocity Distributions

The topic that has received most attention within the group has been the prediction and measurement of velocity distributions in granular materials. In particular, interest has been shown in the formation of stagnant zones in core-flow hoppers. Ideally one would hope that all hoppers are designed to operate in mass flow and indeed reliable techniques are available for the design of such hoppers. However, especially when dealing with agricultural products or products designed for the domestic market, it is frequently necessary to store a new material in an existing hopper which may or may not be ideally suitable.

Many of our measurements have been made by ciné-photography through the transparent walls of plane strain hoppers. For this to be valuable it must be shown that the visible particles are typical of those in the bulk. The early experiments, by Larkin and Ramshaw (Tripos 1971) got off to a bad start because of the use of perspex walls. The particles adjacent to such walls are retarded by as much as 30 % and attempts to reconcile the measured mass flow rate with the volumetric flow rate, predicted by integrating the velocity profile, required unrealistic values of the void fraction. Later work with float glass walls was more successful. Not only were the mass and volumetric flow rates compatible but a more direct confirmation could be obtained as some particles within the bulk could be observed by shining a strong light through the bed. Laohakul [19] (Ph.D. 1978), found that particles adjacent to a float glass wall were never retarded by more than 4 %. Heald and Steen (Tripos 1973) working with a half-cylindrical hopper with a float glass diametral wall came to the same conclusion for the upper part of the hopper. However in the converging flow zone poor agreement was found and they concluded that observations of particles adjacent to a transparent wall was only a useful technique in plane strain.

Most of the experiments have been on free flowing materials such as kale and mustard seed and glass ballotini in the size range 0.5 mm to 2.0 mm. Earlier work by Blair-Fish [11], [20] of the Engineering Department at Cambridge University had used X-radiography of marker particles in sand. He found irregular flow patterns with narrow *dilation zones* being formed periodically between effectively rigid blocks. An explanation of this phenomenon and similar experiments by Cousins [21], also of the Engineering Department, has been sought in terms of plasticity theory. However, no such *dilation zones* were seen in our experiments and the velocities were found to be steady with time and to vary continuously with position. Colledge and Simpson (Tripos 1980) for example injected a succession of coloured particles from a given point and were thereby able to produce visible streamlines that only drifted slowly with time.

Horne's stress calculations, discussed above, show that the material is intersected by a series of stress discontinuities. Any attempt to use plasticity theory will therefore result in the prediction of a velocity field that is discontinuous at least in its first derivative. Not only are the shapes of Horne's predicted stress discontinuities totally different from Blair-Fish's *dilation zones* but no discontinuities of any sort were found in our experiments. This suggests that plasticity theory may not be relevant for the gravity discharge of free flowing materials.

Inspired by the stochastic theories of Litwiniszyn [22] and Mullins [23], a purely kinematic model was developed by Tüzün [24, 25], (Ph.D. 1979). If one considers three adjacent particles as shown in Fig. 1 in which one of the two lower particles is moving faster than the other, the upper particle will clearly have a tendency to move horizontally towards the faster particle. Thus a relationship of the type

$$u = f\left(\frac{\partial v}{\partial x}\right)$$

is to be expected and the simplest non-trivial form

$$u = -B\,\frac{\partial v}{\partial x} \tag{4}$$

was investigated. This gives rise to equations identical to those of Mullins and thereafter the two models are identical. Indeed it is perhaps unfair to describe these as independent models; the argument given above is best thought of as an alternative visualisation of the model which was originally expressed in terms of the hypothetical counter-flow of voids.

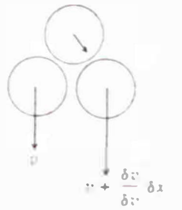

Fig. 1

Coupled with the continuity equation the kinematic model gives rise to the result

$$\frac{\partial v}{\partial y} = B\,\frac{\partial^2 v}{\partial x^2} \tag{5}$$

an equation of the same form as the diffusion equation and for which solutions are known for a great many boundary conditions. For a wide flat bottomed hopper a similarity solution representing flow towards a point sink in a semi-infinite material was found to be appropriate. This takes the form

$$v = \frac{Q}{\sqrt{4\pi B y}}\exp\left\{\frac{x^2}{4By}\right\}. \tag{6}$$

The empirical constant B can therefore be obtained by linear regression on the log of the measured velocities against x^2. B was found to be equal to 2.4 times the particle diameter a result entirely consistent with a model based on the behaviour of adjacent particles.

Equation (6) suggests that there is no truly stagnant zone but that the velocity falls off with increasing horizontal distance to a slow moving zone. Tüzün found that the theoretical streamline representing 99 % of the total flow corresponded closely to the boundary of the apparent stagnant zone. This streamline is a parabola, the width of which depends solely on the particle size and may be thought of as the natural flow boundary in a wide hopper. Tüzün found that for narrower hoppers the same flow boundary occurred up to the point at which it met the wall.

Shearn (Tripos 1974), Tebboth and Walker (Tripos 1976) and Dosekun (Ph.D. 1980) have measured velocities in wedge shaped hoppers. For shallow walls the same velocity distribution occurs as in the flat bottomed hopper but a steep wall that cuts the natural flow boundary causes a change in flow pattern. Dosekun found that the kinematic model predicted the velocity distribution well but he needed larger values of B for steeper walled hoppers.

The basic idea of the kinematic model is that particles move simply by falling into the space vacated by the departing particles below. If this model is correct the velocity distribution will depend solely on factors below the plane of interest. If the more traditional approach of plasticity theory is correct, the velocities will depend on the stresses which, in a material with self weight, will be determined primarily by conditions above the plane of interest. A simple definative test can be made to distinguish between these ideas. If an obstacle is placed within the material, the disturbance to the flow pattern will be above the obstacle if the kinematic theory holds and below if plasticity is valid. The experiments of Davies and Horton [26] (Tripos 1978), and Gorsuch and Warner (Tripos 1979) showed that the disturbance was primarily above the obstacle. In particular the velocity distribution above a square obstacle was found to be quite different from that above a similarly sized equilateral triangle with its apex pointing downwards. That changes in the shape of the underside of the obstacle affect the velocities above the obstacle seems to be a clear indication of the validity of the principle of kinematic modelling even if the formulation of Equation (4) is imperfect.

The kinematic model was also shown to give a good prediction of the motion of the dilation wave on the initiation of discharge from a highly consolidated bed. Within the flowing zone the velocity profile will be given by Equation (6) and the motion of the dilation wave can be found from a mass balance. Buchele and Wynn (Tripos 1980) confirmed experimentally that Tüzün's value for B correctly predicted the rate of opening up of the flowing zone in a wide hopper.

The kinematic model does not give a good prediction of the velocity adjacent to a rough surface. Laohakul [19] measured the velosity distribution and thickness of these boundary or shear layers. He found that the thickness of the layers was always about 5 particle diameters and seemed to be independent of velocity or wall roughness. This suggests that an explanation in terms of geometric and not dynamic effects is to be sought.

Whilst the kinematic model was found to give a good description of the gravity flow of coarse free flowing material, the behaviour of particles of diameter less than 500 μm was quite different. For these materials the flow is irregular and Tüzün [25] found dilation zones similar to those observed by Blair-Fish [20]. In neither set of experiments did the dilation zones lie along the stress discontinuities predicted by Horne [9] and it is therefore difficult to explain their presence by plasticity theory. Since the breakdown of the kinematic model occurs at just the particle size at which interstitial pressure gradients begin to affect the flow rate, it is tempting to attribute the formation of these zones to interstitial pressure effects. It is hoped to check this by conducting experiments under reduced pressure.

Velocity measurements in three dimensions are much more difficult and several projects have been concerned with measuring residence time distributions. Whilst these cannot always be inverted to give the velocity distribution they can at least be compared with the residence time distribution predicted from some theoretical velocity profile. Woodward (Tripos 1972) measured the discharge time for particles placed at known positions in the initial packed bed and both Dunlop and Thomas (Tripos 1973) and Adu and Brown (Tripos 1977) made more conventional residence time measurements by the injection of coloured tracers into the feed to a continuously discharging hopper. More recently Smallwood and Thorpe (Tripos 1980) have released steel ball bearings at points within a flowing bed by dropping them down a thin vertical tube.

The time of passage was measured electrically by having detecting coils round the tube and at the orifice. From these, contours of constant discharge time can be plotted for the whole hopper. This distribution can be inverted to give the velocity profile though the numerical problems are formidable. Despite this they obtained good agreement with Tüzün's results for a plane strain hopper and also some preliminary results for cylindrical systems. It is hoped to continue this work next year.

6. Future Work

The shortage of research students makes it likely that there will be fewer major projects in progress than in recent years. However the award of a post-doctoral grant to Dr. Tüzün has enabled us to embark on a more detailed study of the effect of obstacles. It is hoped to predict the stress distribution by the methods established by Horne and to measure the resulting wall stresses. It is planned to measure the velocity distribution at the same time and thereby gain information about the applicability of plasticity theory and kinematic modelling for this situation.

By contrast with research student numbers, undergraduate numbers are increasing rapidly and I foresee the possibility of having several Tripos projects for many years to come. The following four projects are planned for the coming year (1980—81):

6.1 Continuation of the Residence Time Measurements by the Ball Bearing Technique of Smallwood and Thorpe

This will probably be done in a conical hopper where the contours of constant discharge time should be more regular than for the cylindrical hopper with its stagnant zones.

6.2 Continuation of Bunt and Smith's Work on Interstitial Pressure Profiles

In particular we hope to develop relationships between the pressure gradient at the orifice and the overall pressure drop in a cylindrical bunker and to make allowance for inertial effects.

6.3 Use of Kinematic Modelling to Predict the Coning of the Top Surface of the Material in Batch Discharge

Since the kinematic model predicts the velocities adequately it should also predict the motion of the particles on the top surface until the slope of that surface becomes steep enough for cascading flow to develop.

6.4 Study of the Initial Transient in Cylindrical Bunkers

Buchele and Wynn have studied the motion of the dilation wave separating the flowing material from that at the original voidage. Their work was in plane strain and it is hoped to extend this to cylindrical systems using techniques similar to those of Woodward.

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