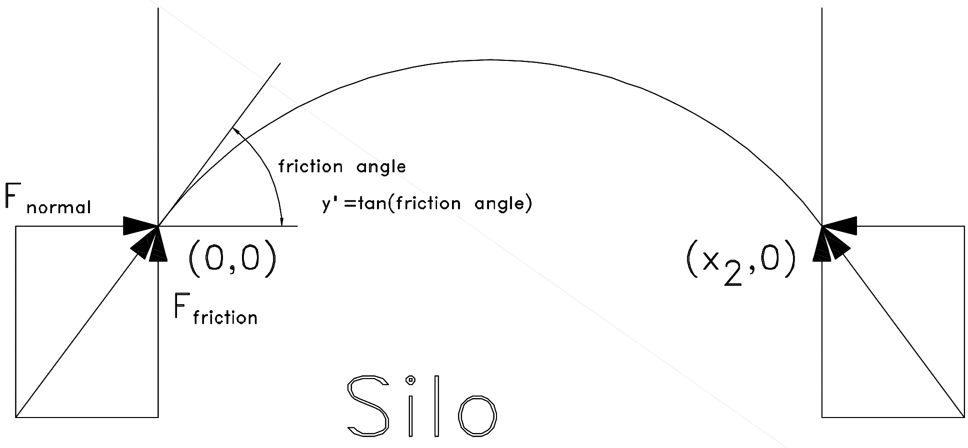
Catenary equation.

A catenary has the same mathematical formula as an arch, whereby the compression forces in the arch are replaced by the tension in a chain or a belt

MATHEMATICAL EQUATION OF AN ARCH



General equation of an arch (as an inverse catenary):

$$y = \lambda * \cosh\left(\frac{x}{\lambda} + C\right) + a \qquad y' = \sinh\left(\frac{x}{\lambda} + C\right)$$

For $x=0$ → $y=0$

$$y'_{(0,0)} = f_{wall} = \tan(\varphi_{wall}) = \sinh(C)$$

from: $f_{wall} = \sinh(C)$ follows: $\frac{e^C - e^{-C}}{2} = f_{wall}$

resulting in : $e^C - e^{-C} - 2 * f_{wall} = 0$

In case: $x = e^C \quad x - \frac{1}{x} = 2 * f_{wall}$

$$x^2 - 2 * x * f_{wall} - 1 = 0$$

$$x = \frac{2 * f_{wall} + \sqrt{(2 * f_{wall})^2 + 4}}{2}$$

then: $$C = \ln(x) = \ln\left(\frac{2 * f_{wall} + \sqrt{(2 * f_{wall})^2 + 4}}{2}\right) \qquad \textbf{(1)}$$

For $x = 0$ → $y = 0$

$$0 = \lambda * \cosh\left(\frac{0}{\lambda} + C\right) + a \quad \rightarrow \quad a = -\lambda * \cosh(C) \qquad \textbf{(2)}$$

For $x = x_2$ → $y = 0$

in case : $Q = (x_2 + C)$

then : $\sinh(Q) = -y' = -f_{wall}$ $\qquad f_{wall} = \sinh(C)$

$$e^Q - e^{-Q} = -2 * \sinh(C)$$

$$e^Q - e^{-Q} = -e^C + e^{-C} = e^{-C} - e^C$$

resulting in: $Q = -C$

$$-C = \left(\frac{x_2}{\lambda} + C\right)$$

$$\lambda = -\frac{x_2}{2 * C} \qquad \textbf{(3)}$$

Using the formulas, the equation for the arch can be calculated as follows:

- calculate $C = \ln\left(\frac{2 * f_{wall} + \sqrt{(2 * f_{wall})^2 + 4}}{2}\right)$
- calculate $\lambda = -\frac{x_2}{2 * C}$
- calculate $a = -\lambda * \cosh(C)$
- calculate any point on the arch with: $y = \lambda * \cosh\left(\frac{x}{\lambda} + C\right) + a$